Lection 2

Excitons in external fields (1h)
(bulk exciton in a magnetic field, Zeeman effect, diamagnetic shift; exciton in an electric field, Stark effect, the exciton in QW in external fields, confined stark effect; case of the degenerate valence band; Pockels effect)

Exciton in magnetic fields

(simple band structure)

In magnetic field we have to change $\mathbf{p} \to \mathbf{p} + \frac{e}{c}\mathbf{A}$ obtain

$$\left[-\frac{\hbar^2\nabla_e^2}{2m_e} - \frac{\hbar^2\nabla_h^2}{2m_h} - \frac{e^2}{\varepsilon r_{eh}} - \frac{ie\hbar}{m_e c}A(\mathbf{r}_e)\nabla_e + \frac{ie\hbar}{m_h c}A(\mathrm{r}_h)\nabla_h + \frac{e^2}{2m_e c^2}A^2(\mathbf{r}_e) + \frac{e^2}{2m_h c^2}A^2(\mathbf{r}_h)\right]\Psi(\mathbf{r}_e,\mathbf{r}_h) =$$
$$= E\Psi(\mathbf{r}_e,\mathbf{r}_h)$$

Relative motion and center of mass motion

$$-i\hbar\nabla_e = \left(-i\hbar\nabla + \frac{e}{c}\vec{A}(\vec{r}) + \alpha\hbar\vec{Q}\right) \qquad -i\hbar\nabla_h = \left(i\hbar\nabla + \frac{e}{c}\vec{A}(\vec{r}) + \beta\hbar\vec{Q}\right)$$

Take the trial function in the form $\Psi(\vec{R},\vec{r}) = \exp\left\{i\left[\vec{Q} - \frac{e}{\hbar c}\vec{A}(\vec{r})\right]\vec{R}\right\}F(\vec{r})$

For $F(\vec{r})$ get equation

Relative motion — Angular momentum — diamagnetic — Magneto stark effect

$$\left[-\frac{\hbar^2\nabla^2}{2\mu}-\frac{e^2}{\varepsilon r}+\frac{ie\hbar}{c}\left(\frac{1}{m_e^*}-\frac{1}{m_h^*}\right)\vec{A}(\vec{r})\cdot\vec{\nabla}+\frac{e^2}{2\mu c^2}\vec{A}^2(\vec{r})-\frac{2e\hbar}{\left(m_e^*+m_h^*\right)c}\vec{A}(\vec{r})\cdot\vec{Q}\right]F=$$

$$=\left[E-\frac{\hbar^2Q^2}{2\left(m_e^*+m_h^*\right)}\right]F$$

Angular momentum **L** $\quad \mathbf{A}(\mathbf{r})\cdot\nabla=-2i\hbar\,\mathbf{H}\cdot\mathbf{L}$

Orbital Zeeman effect $\sim \left(\frac{1}{m_e^*}-\frac{1}{m_h^*}\right)$ and can be small

usually $m_h^*\gg m_e^*$

Contrary to spin magnetism, here we have effective mass

$$\frac{m_0}{m_e^*}\sim 10 \quad \Rightarrow \quad \mu_{spin}=\frac{e}{mc},\quad \mu_{orbit}\propto 10\mu_{spin}$$

The term $\vec{A}\cdot\vec{Q}$ Is magneto Stark effect

$$\frac{2e\hbar}{(m_e+m_h)c}\mathbf{A}(\mathbf{r})\cdot\mathbf{Q}=\frac{e\hbar}{Mc}[\mathbf{Q}\times\mathbf{B}]\cdot\mathbf{r}=\frac{e}{c}[\mathbf{v}\times\mathbf{B}]\cdot\mathbf{r}$$

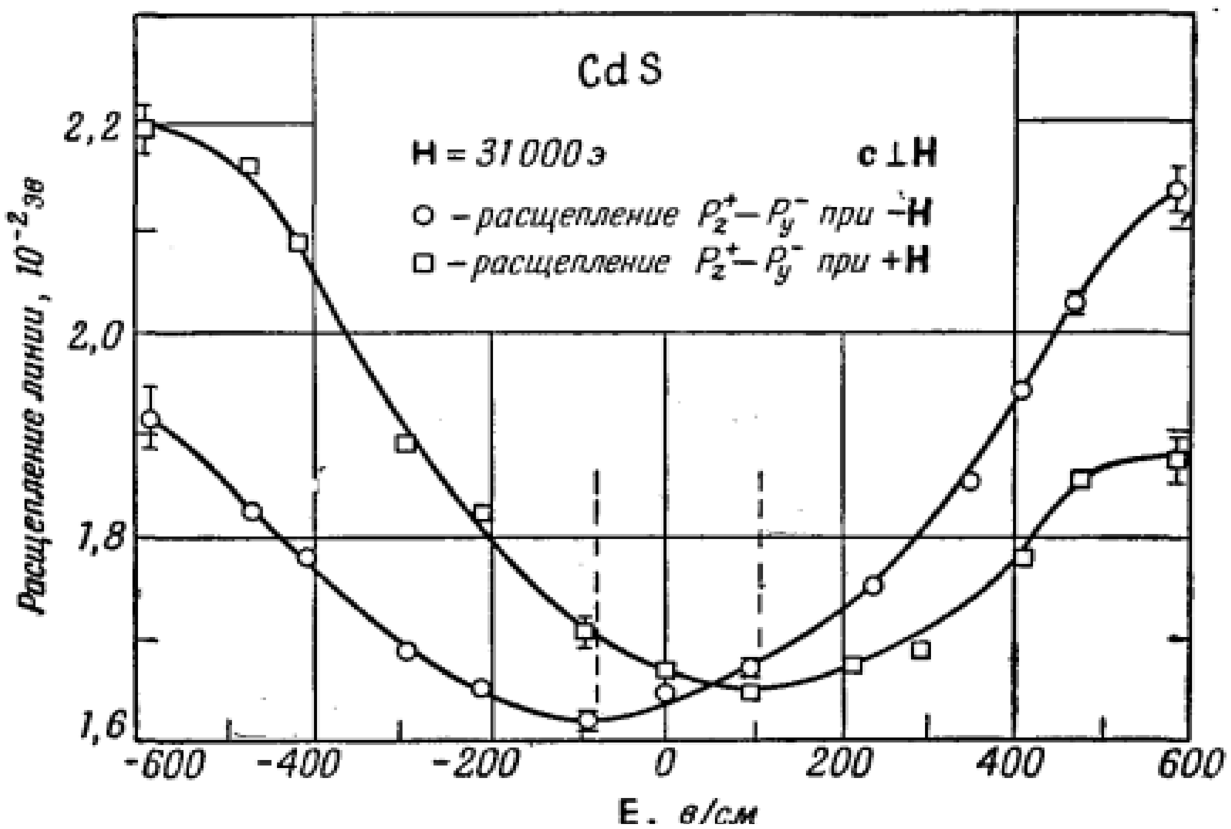
$\frac{e}{c}[\mathbf{v}\times\mathbf{B}]$ Is the Lorentz force

It can be compensate by an electric field $\mathbf{E}$ because

$$\left(\mathbf{E}+\frac{e}{c}[\mathbf{v}\times\mathbf{B}]\right)\cdot\mathbf{r}$$

The correction to energy is in the second order perturbation

$\propto Q^2$ and $\propto B^2$ Corrections to effective mass and diamagnetic shift



Magneto-Stark effect in CdS crystal in magnetic fields

(Thomas Hopfield Phys. Rev. Lett. 5, 505 (1960))

In magnetic fields

Additionally to spin magnetism

1). Orbital magnetism on *p* states

2). Diamagnetic shift

3). Magneto-Stark effect

Exciton in magnetic fields

(cubic Td crystal)

$$H_{exc} = H_{ex}(0) + H_{ex}(B) + H_{ex}(K) + H_{ex}(K \cdot B) + H_{ex}(K^2) + H_{ex}(B^2) + \ldots$$

Here: $H_{ex}(0)$ exciton Hamiltonian in zero field and zero K

$$H_{ex}(B) = g_e \mu_B (\boldsymbol{\sigma} \cdot \mathbf{B}) - 2\mu_B \left[\tilde{k} (\mathbf{J} \cdot \mathbf{B}) + \tilde{q} \left(B_x J_x^3 + B_y J_y^3 + B_z J_z^3 \right) \right]$$

linear in magnetic field contribution

$$H_{ex}(K) = C \left[K_x J_x \left(J_y^2 - J_z^2 \right) + K_y J_y \left(J_z^2 - J_x^2 \right) + K_z J_z \left(J_x^2 - J_y^2 \right) \right]$$

linear in wave vector contribution

$$H_{ex}(K\,B) = A_1 \left[B_x H_x \left(J_y^2 - J_z^2 \right) + c.p. \right] + A_2 \left([\mathbf{B}\mathbf{K}]_x \left\{ J_y \cdot J_z \right\} + c.p. \right)$$

bilinear in magnetic field and wavevector term

$$H(K^2) = \frac{\alpha^2}{2m_e} K_z^2 I + \frac{\beta^2}{2m_0} \left(\gamma_1 + \frac{5}{2}\gamma \right) K_z^2 I - \frac{\gamma}{m_0} \beta^2 \left(K_z J_z \right)^2$$

exciton in the absence of magnetic field

Exciton in magnetic fields

(complex band structure)

Exciton Hamiltonian

$$H = \hbar^2 \mathbf{K}_e^2 / 2m_e - \frac{\hbar^2}{2m_0}[(\gamma_1 + \frac{5}{2}\gamma)\mathbf{K}_h^2 \mathbf{I} - 2\gamma(\vec{J}\mathbf{K}_h)^2] - \frac{e^2}{\kappa|\vec{r}_e - \vec{r}_h|}$$

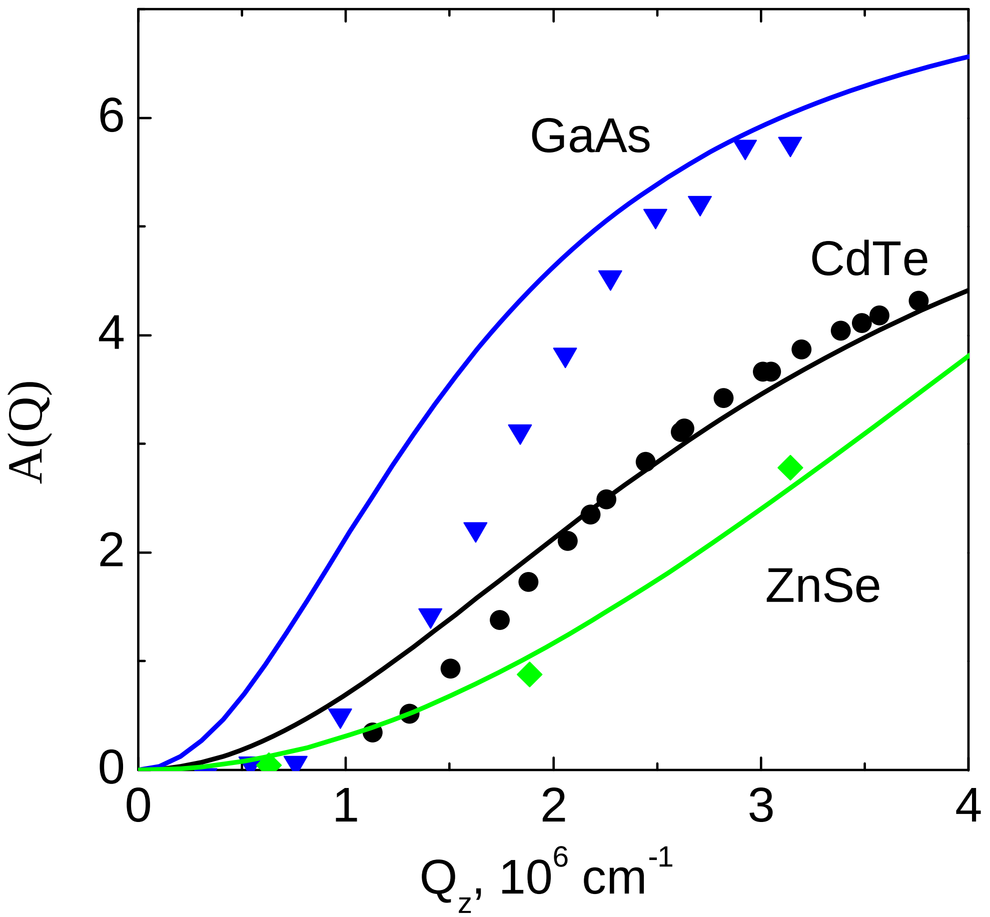
$$-i\hbar\nabla_e = \left(-i\hbar\nabla + \frac{e}{c}\mathbf{A}(\vec{r}) + \alpha\hbar\mathbf{Q}\right) \qquad -i\hbar\nabla_h = \left(i\hbar\nabla + \frac{e}{c}\mathbf{A}(\vec{r}) + \beta\hbar\mathbf{Q}\right)$$

Because of the complex band structure we can not separate internal motion and center of mass motion in Faraday geometry we always have the term:

$$\frac{\beta\hbar\gamma}{m_0}[(\mathbf{p}_x + \frac{e}{c}A_x)J_x + (\mathbf{p}_y + \frac{e}{c}A_y)J_y](Q_z J_z)$$

This term mixed 1*s* ground state and all *p* states of the internal motion and leads to two effects for <u>moving exciton</u>:

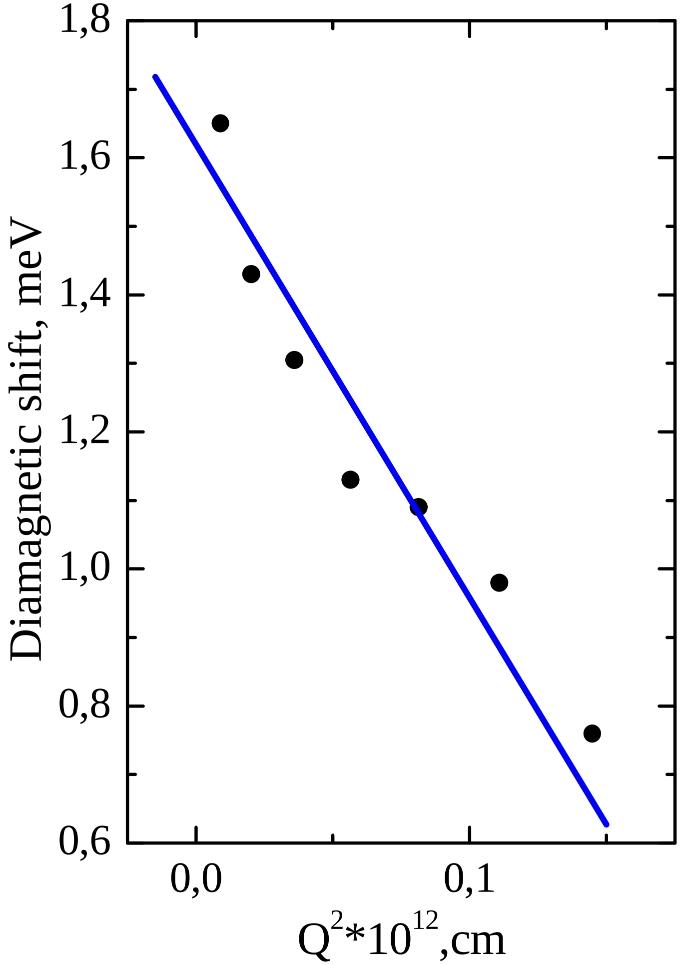
1) Increase exciton Zeeman splitting



$$\mathcal{E}_{1S}^{(2)}(Q,B) = 2SB_z Q_z^2 (7J_z - 4J_z^3)$$

$$S = \left(\frac{\gamma}{m_0}\right)^2 \hat{\beta}^2 \left(\frac{e\hbar}{c}\right)\left(\frac{\hbar^2}{2}\frac{1}{Ry^*}\right)\sum_{n=2}^{\infty}\frac{\langle 1S|r/a_B|nP\rangle\langle 1S|a_B\nabla|nP\rangle}{1-1/n^2+\Delta(Q)/Ry^*}$$

2) Decreasing of diamagnetic shift



$$\varepsilon_d(B,Q) = D_1 Q_z^2 B^2 \mathbf{I}$$

$$D_1 = \frac{3}{4}\left(\frac{\gamma}{m_0}\right)^2 \hat{\beta}^2 \left(\frac{e\hbar}{c}\right)^2 \left(\frac{1}{Ry^*}\right) a_B^2 \sum_n \frac{\left|\left\langle 1S \left| r / a_B \right| nP_y \right\rangle\right|^2}{1 - 1/n^2 + \Delta(Q)/Ry^*}$$

These effects are present in narrow wells also and in QDs and in NWs

In the case of degenerate valence band

Due to mixing of internal and center of mass motion

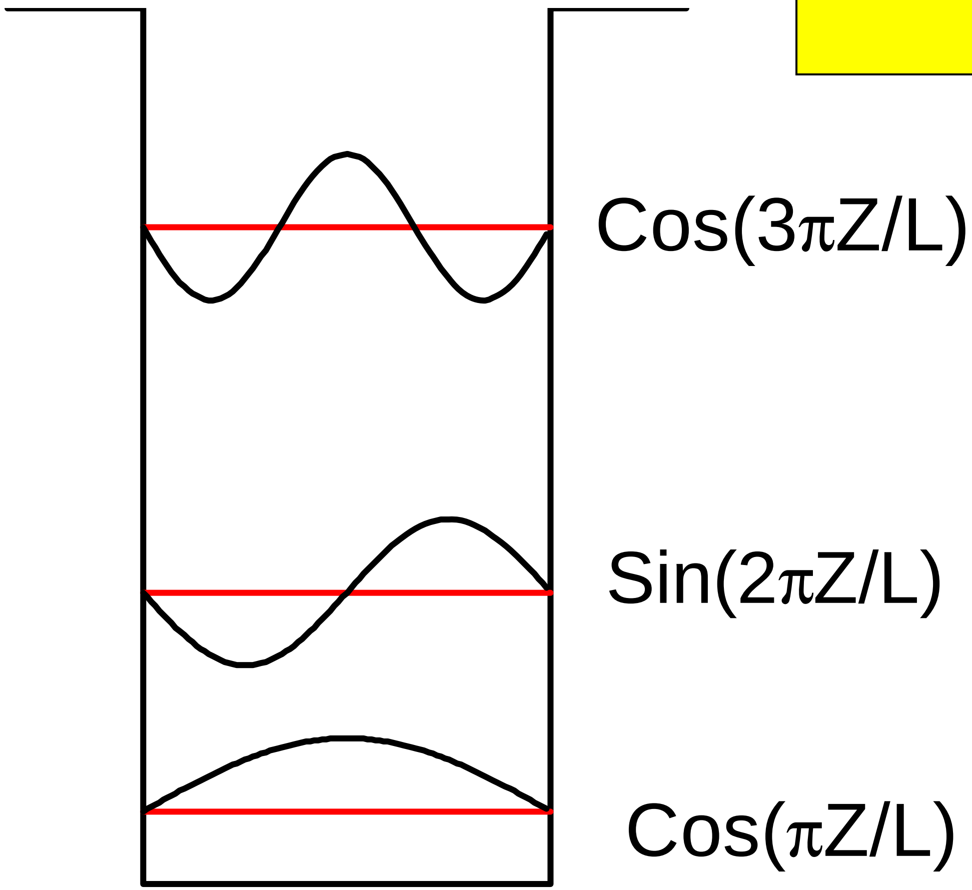
1). Zeeman splitting

2). Diamagnetic shift

depend on the center of mass energy

Effect of parity in the exciton spectra

we have even and odd exciton quantized states



$K_N L = \pi N$

For QW having with $L \approx \lambda/2$ the only even exciton states are optically active

For QW having with $L \approx \lambda$ the only odd exciton states are optically active

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QUANTIZATION OF EXCITONIC POLARITONS IN CdTe-CdZnTe DOUBLE HETEROSTRUCTURES

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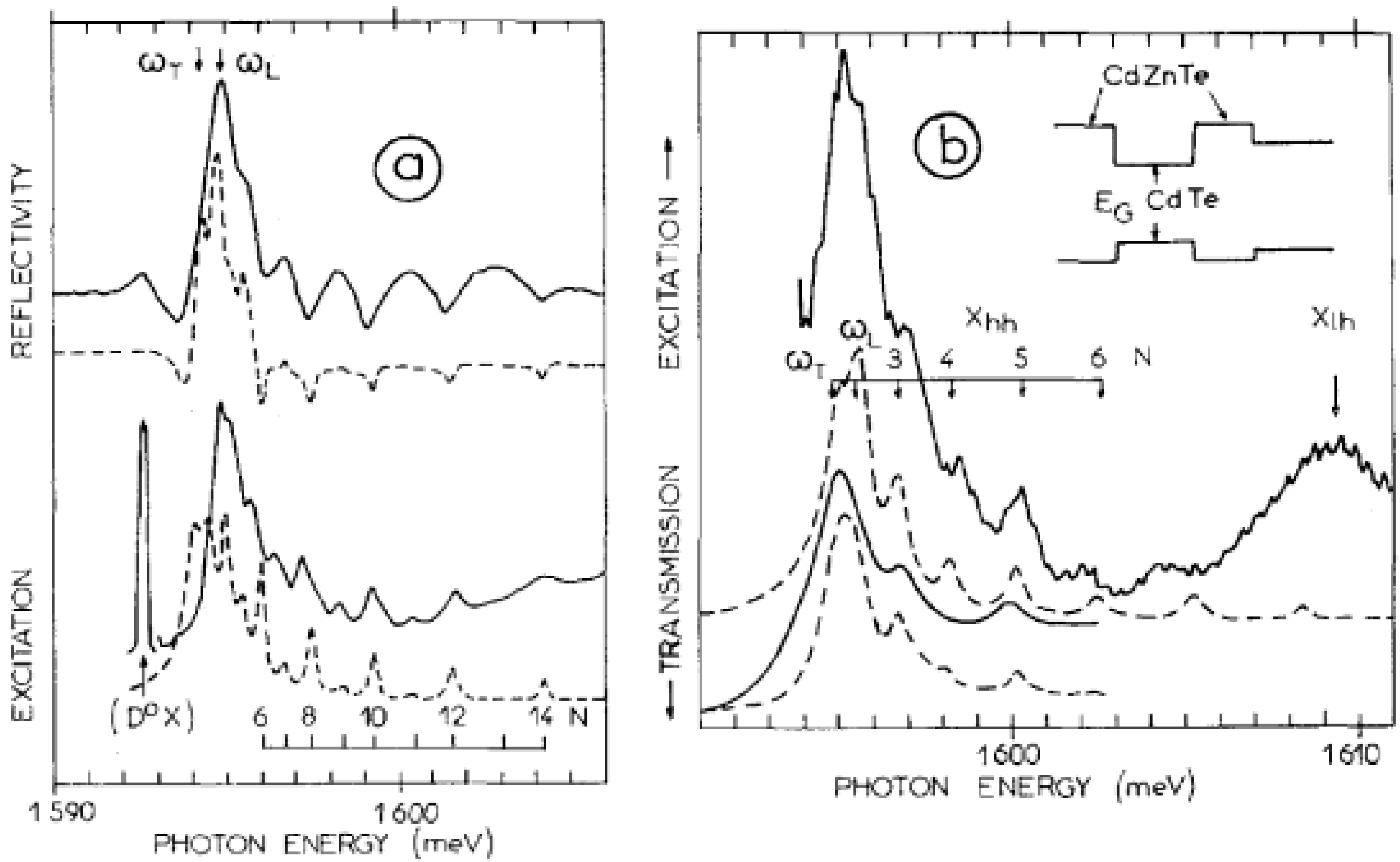


Faraday geometry

Linear in wave vector and magnetic field contribution in bulk T_d

$$H_{ex}\left(KH\right)=B_1\left[K_xH_x\left(J_y^2-J_z^2\right)+c.p.\right]$$

In D_{2d} similar

consider the first term

$$B_1K_zH_z\times\begin{bmatrix}0&0&\sqrt{3}&0\\0&0&0&\sqrt{3}\\\sqrt{3}&0&0&0\\0&\sqrt{3}&0&0\end{bmatrix}$$

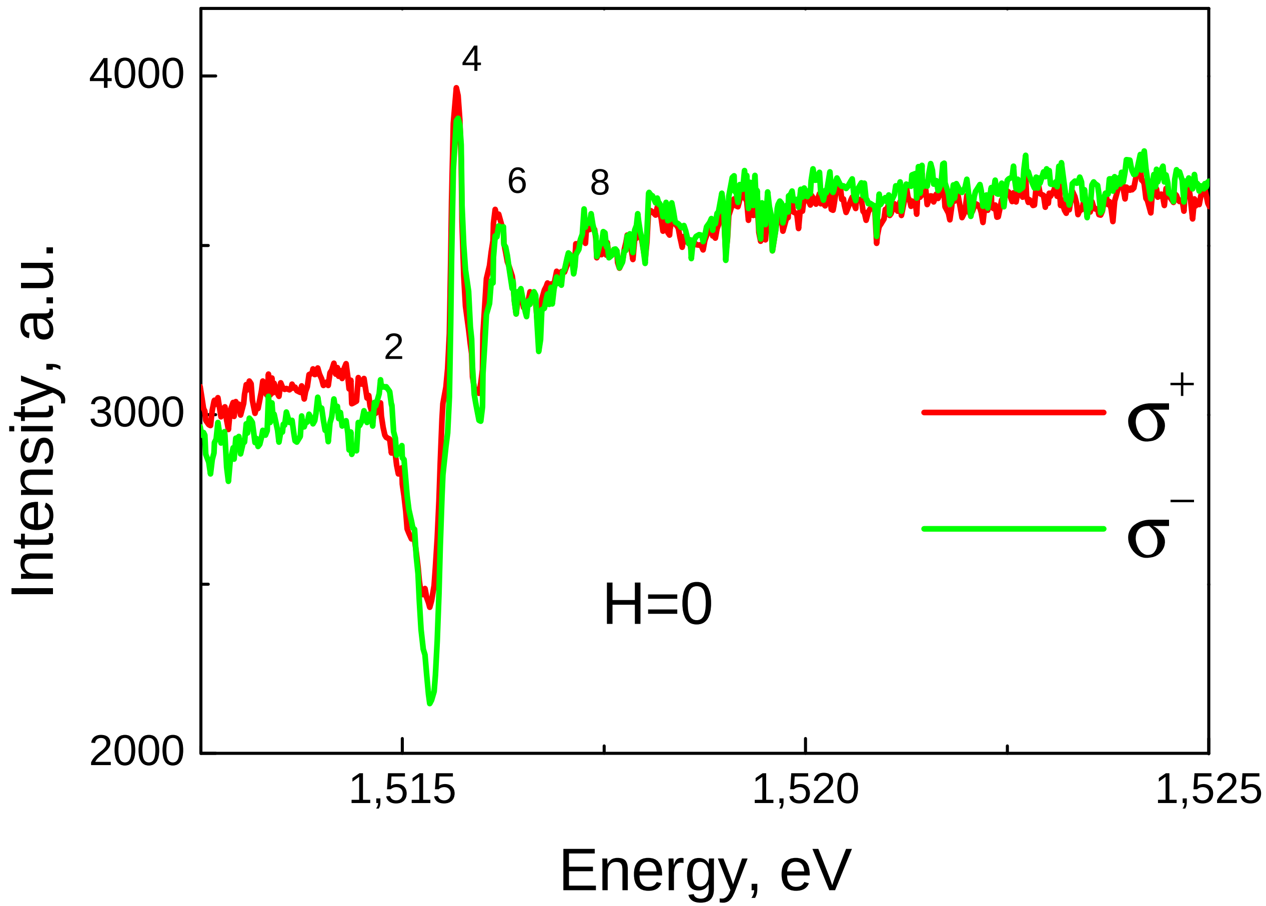
Mixing of *HH* and *LH* excitons

$$(\mathcal{E}_{HH}^0 - \hbar\omega)\left|x_{HH}\right\rangle + i\mu g_{HH} H_z \left|y_{HH}\right\rangle = d_x E_x + BH_z \frac{1}{3}\frac{d_x}{\left|\mathcal{E}_{LH}^0 - \mathcal{E}_{HH}^0\right|} i\nabla_z E_x$$

$$(\mathcal{E}_{HH}^0 - \hbar\omega)\left|y_{HH}\right\rangle - i\mu g_{HH} H_z \left|x_{HH}\right\rangle = d_y E_y - BH_z \frac{1}{3}\frac{d_y}{\left|\mathcal{E}_{LH}^0 - \mathcal{E}_{HH}^0\right|} i\nabla_z E_y$$

Reflectivity spectrum taken from GaAs/AlGaAs QW at zero magnetic field

Incident light is linearly polarized in (100), circular polarized component is analyzed

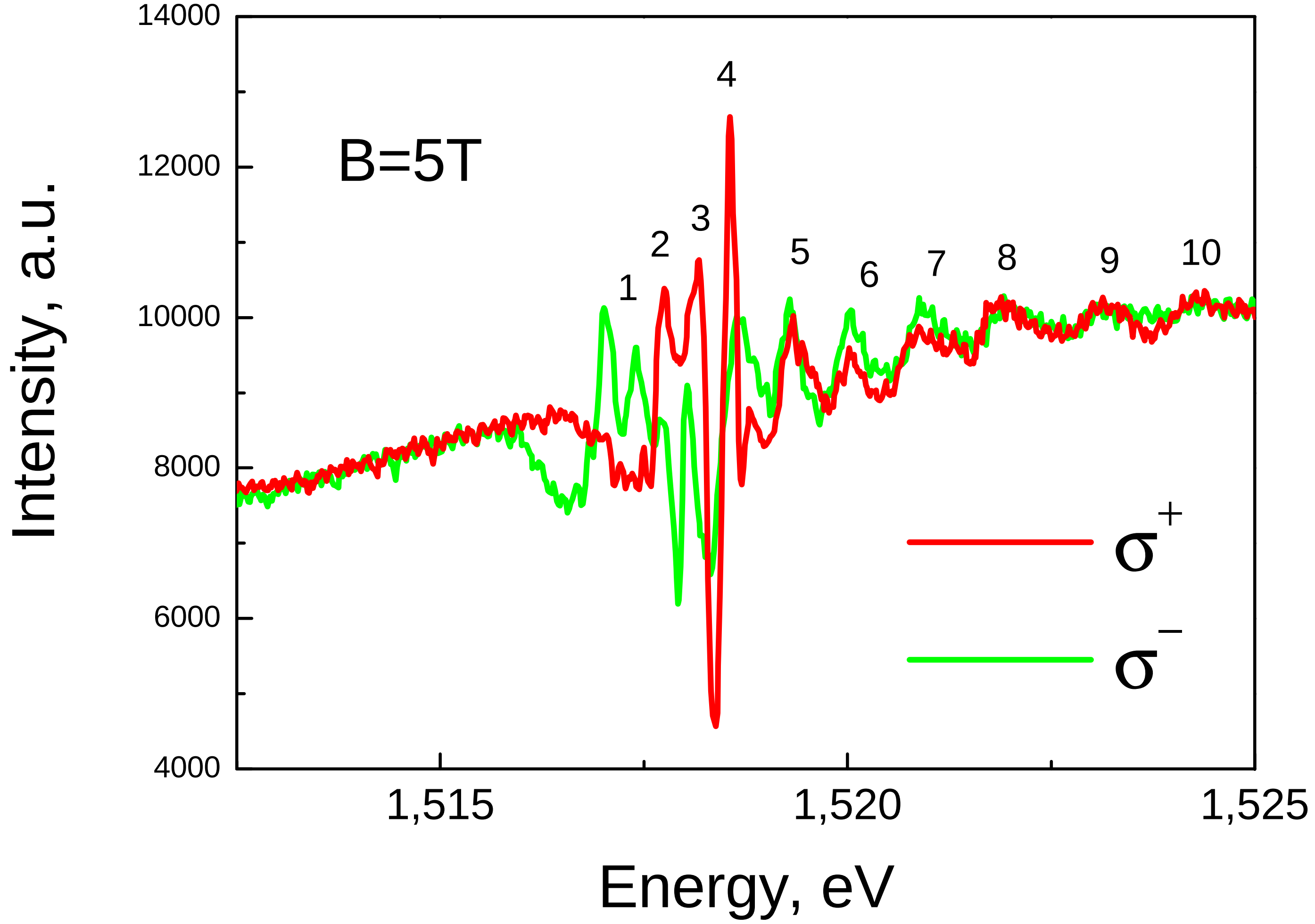


L=300nm

Only even states are present in this spectrum

Redistribution of exciton oscillator strength between odd and even states

Incident light is linearly polarized,
circular polarization is analyzed



Voigt geometry

Cubic crystal

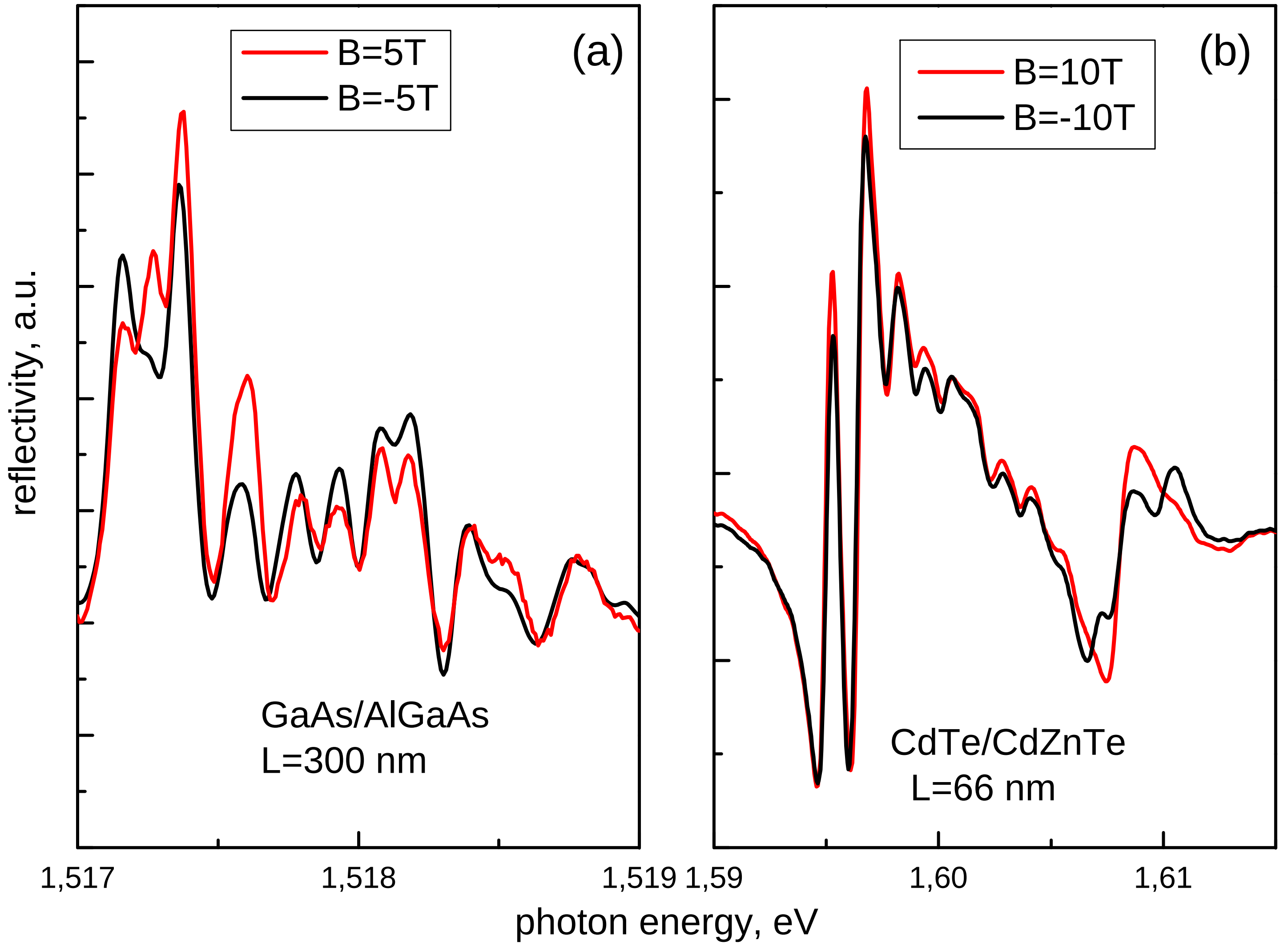
$$B_2\left(\left[\vec{H}\vec{K}\right]_x\left\{J_y\cdot J_z\right\}+\left[\vec{H}\vec{K}\right]_y\left\{J_x\cdot J_z\right\}\right)$$

$$\left\{J_yJ_z\right\}=\begin{bmatrix} 0 & -i\sqrt{3} & 0 & 0 \\ i\sqrt{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & i\sqrt{3} \\ 0 & 0 & -i\sqrt{3} & 0 \end{bmatrix}$$

$$\left\{J_xJ_z\right\}=\begin{bmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{3} \\ 0 & 0 & -\sqrt{3} & 0 \end{bmatrix}$$

Voigt geometry

Magnetic field inversion effect



Voigt geometry

Wurzite crystal

For the conductivity band

$$H_{\Gamma_7}(\mathbf{k},\mathbf{H}) = \left(E^0_{\Gamma_7} + \frac{\hbar^2 k_z^2}{2m_{II}} + \frac{\hbar^2 k_\perp^2}{2m_\perp} \right)\hat{I} + a[\mathbf{H}\times\mathbf{k}]_z \hat{I} + b(\sigma_x k_y - \sigma_y k_x) + \frac{1}{2}\mu(g_{II} H_z \sigma_z + g_\perp(\mathbf{H}_\perp \cdot \boldsymbol{\sigma}_\perp))$$

For the valence band

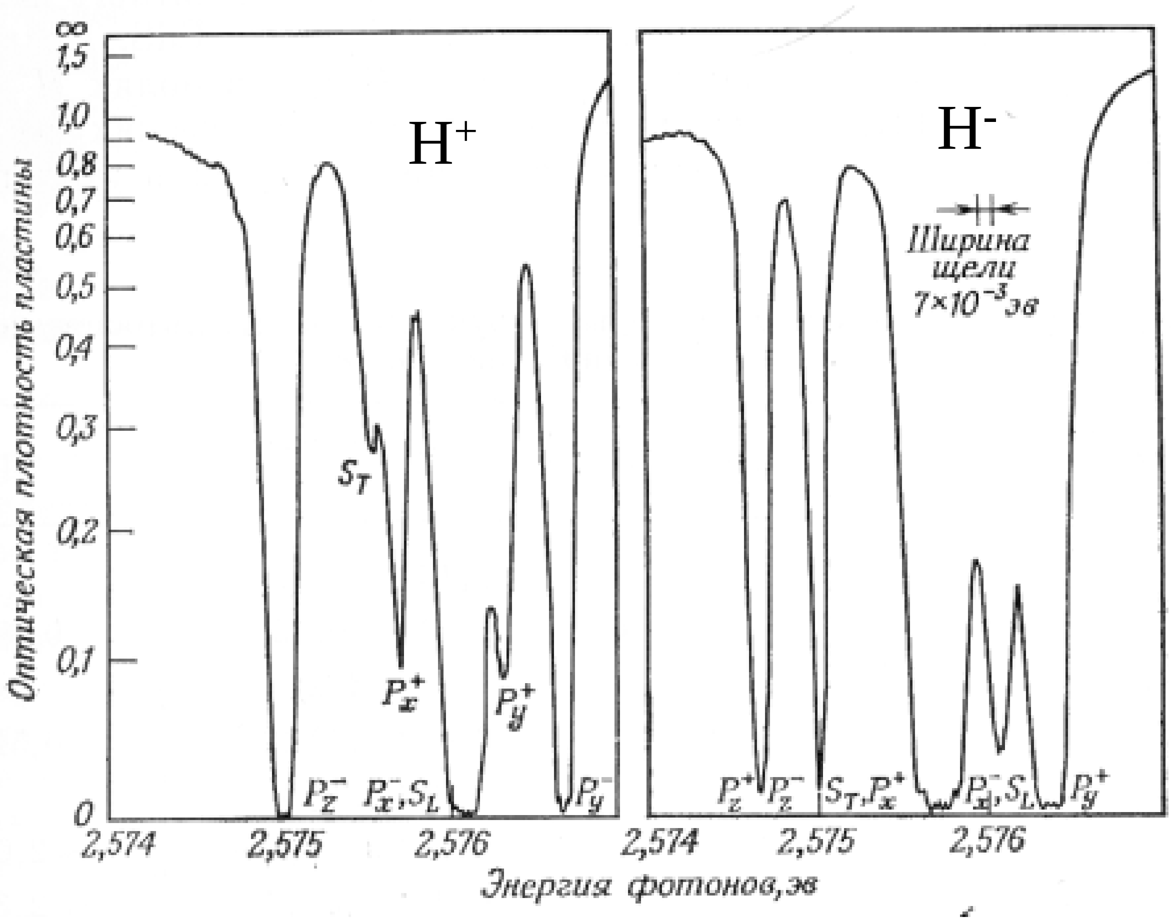
$$H_{\Gamma_9}(\mathbf{k},\mathbf{H}) = \left(E^0_{\Gamma_9} - \frac{\hbar^2 k_z^2}{2m_{II}} - \frac{\hbar^2 k_\perp^2}{2m_\perp} \right)\hat{I} + a_1[\mathbf{H}\times\mathbf{k}]_z \hat{I} + d_1\sigma_x(k_x^3 - 3k_x k_y^2) + d_2\sigma_y(3k_x^2 k_y - k_y^3) + \frac{1}{2}\mu g_{II} H_{II}$$

optical transitions

$$\left| D_v + \kappa_y Q_v \right|^2 \quad \text{and} \quad \left| D_v - \kappa_y Q_v \right|^2$$

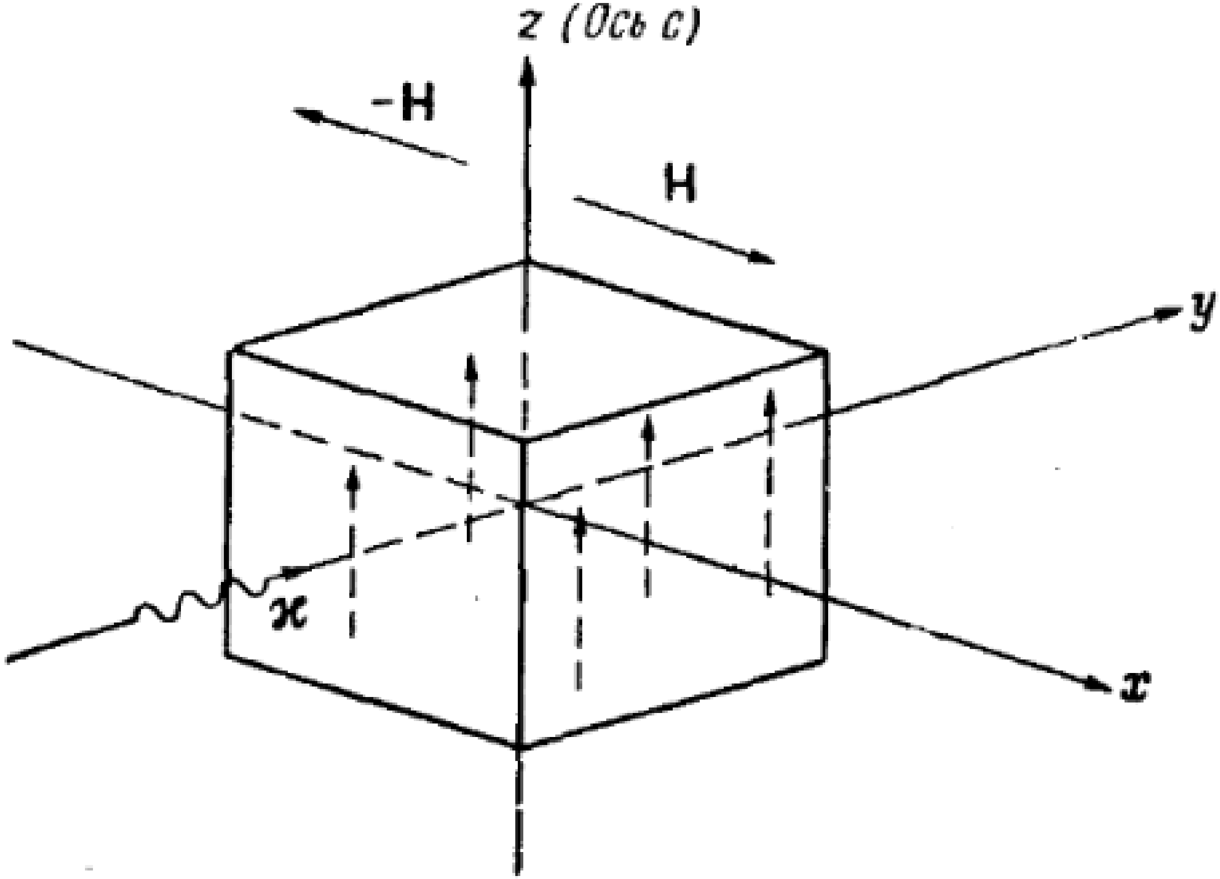
Effect of magnetic field inversion

$$\varepsilon(\omega, K, H) \neq \varepsilon(\omega, -K, H)$$
$$\varepsilon(\omega, K, H) = \varepsilon(\omega, -K, -H)$$



Transmission spectra of CdS in magnetic field of 3.1T, H is perpendicular to C_6 axis

Effect of inversion of magnetic field



In the structures without inversion symmetry

1). Effect of “parity” (due to nonreciprocal magnetic field induced birefringence)

2). “Effect of inversion of magnetic field”

Quantum confined Stark effect

Lateral electric field $F \parallel (\mathrm{x}, \mathrm{y})$

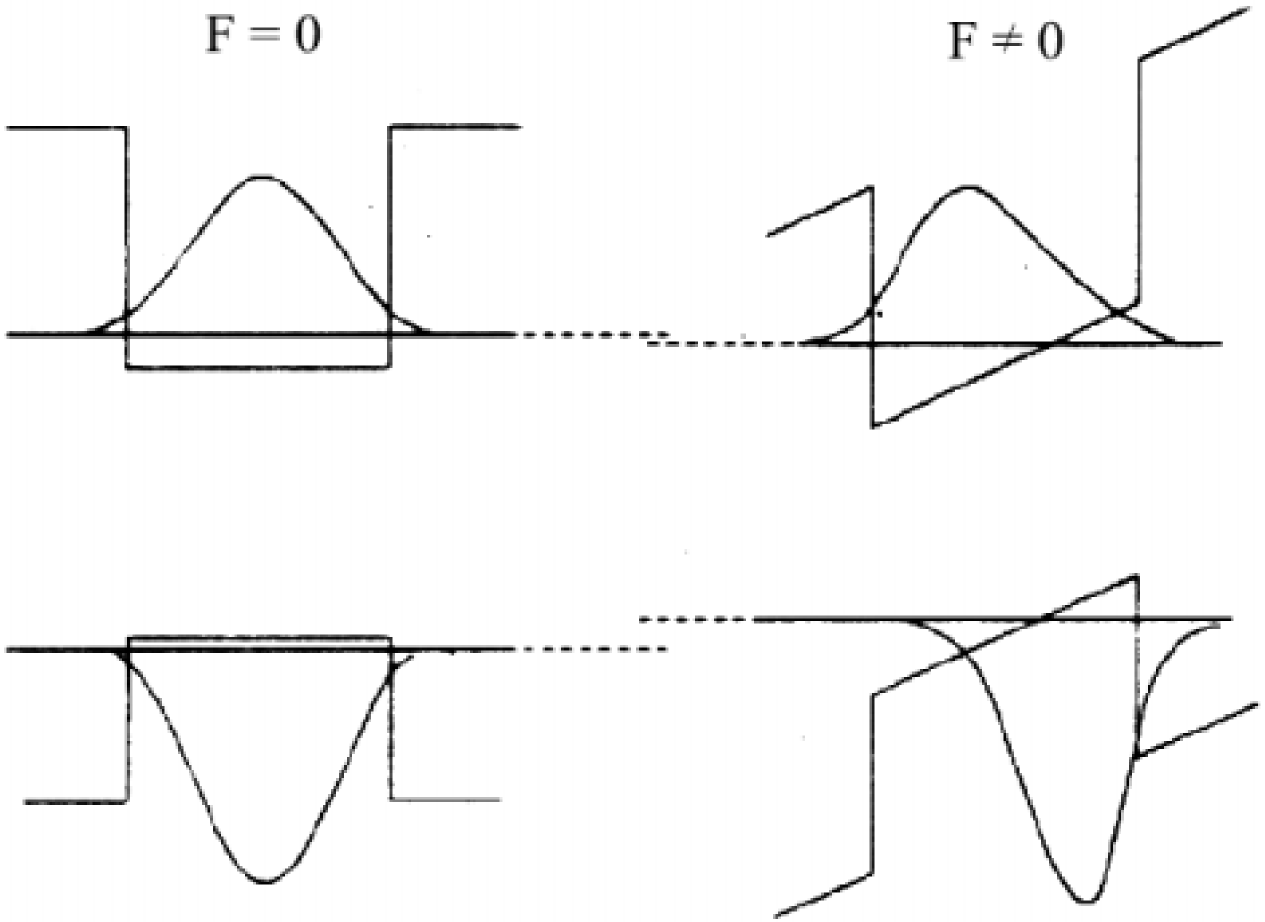
Two effects: 1) current, 2) dissociation of the exciton

Broadening of the exciton level $-\ln\Gamma \propto \frac{\Delta x}{a_B} \propto \frac{E_B}{|e| F_{\parallel} a_B}$

Transverse field $F \perp (\mathrm{x}, \mathrm{y})$

No dissociation, but Stark shift

$$\Delta E_{exc} = \delta E_{e1} + \delta E_{h1} - \delta\varepsilon \qquad \delta E_1 = -\sum_{n\neq 1} \frac{(eF\, z_{n1})^2}{E_n - E_1} \propto -0.01 \frac{(eFd)^2}{E_1}$$


F = 0
F ≠ 0

Stark effect in superlattice

In the tight binding model we have equation

$$IC_{n-1} + E_0 C_n + IC_{n+1} = EC_n$$

E - Energy, I - transfer integral

For the Bloch solutions: $C_n = \exp(iK_z dn)$ miniband:

$$E(K_z) = E_0 + 2I\cos K_z d$$

The miniband width equal $\Delta = 4I$

Effective mass $M = \dfrac{\hbar^2}{2|I|d^2}$

In the presence of electric field tight binding equation

$$I\left(C_{n-1}+C_{n+1}\right)+\left(E_0+|e|Fdn\right)C_n=0$$

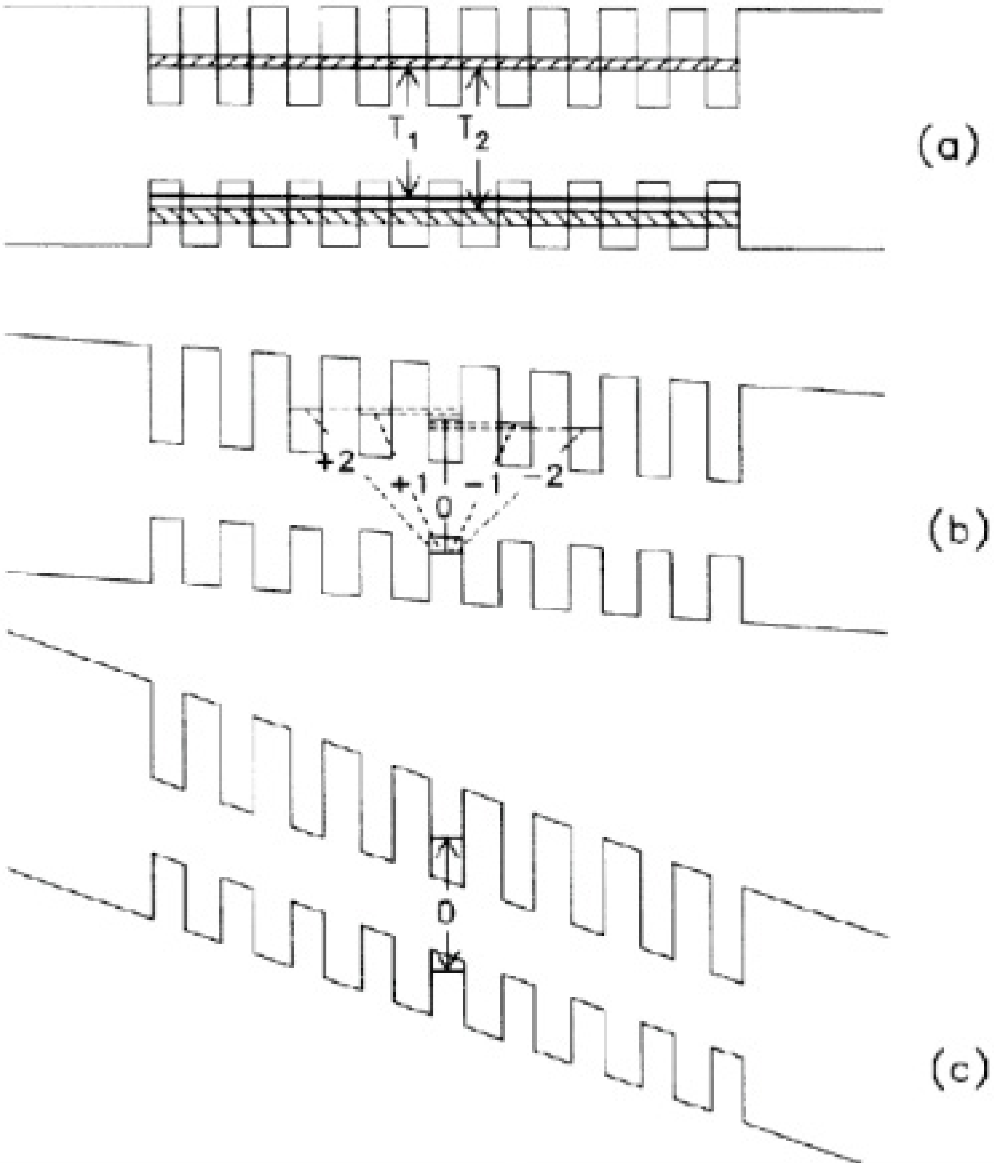
The electric field just shifts the energy E_0

When the detuning $|e|Fd$ becomes bigger the miniband width Δ

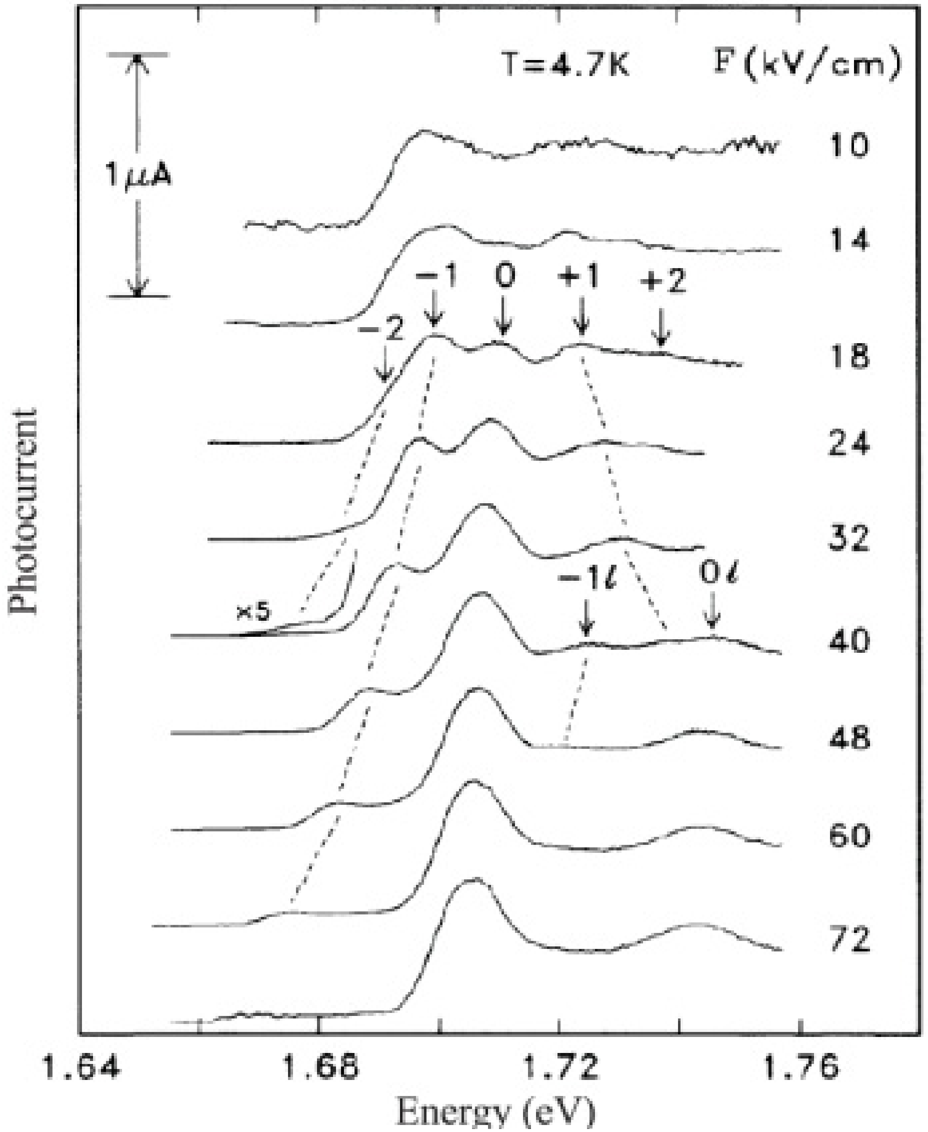
LOCALIZATION

Localization length $L\approx\dfrac{\Delta}{|e|F}$

Wannier Stark localization



Wannier Stark localization



In superlattices

Wannier-Stark localization

Quantum confined Pockels effect

(Linear in electric field Birefringence)

Dielectric function

$$\varepsilon_{ij}(\omega,\mathbf{k},\mathbf{E}) = \varepsilon_{ij}(\omega) + i\gamma_{ijl}(\omega)k_l + A_{ijl}(\omega)E_l + B_{ijlm}(\omega)k_l k_m + C_{ijlm}(\omega)E_l E_m + D_{ijlm}(\omega)k_l E_m + \ldots.$$

$\varepsilon_{ij}(\omega)$ normal frequency dispersion

$\gamma_{ijl}(\omega)k_l$ natural optical activity (gyrotropy)

$A_{ijl}(\omega)E_l$ Pockels effect

$B_{ijlm}(\omega)k_l k_m$ spatial dispersion due to exciton motion

$C_{ijlm}(\omega)E_l E_m$ Kerr effect

$D_{ijlm}(\omega)k_l E_m$ electric field induced gyrotropy

1). Bulk mechanism

$$\varepsilon_{ij}\left(\omega,\mathbf{E},\mathbf{K}\right)=\varepsilon_{ij}^{0}\left(\omega,\mathbf{K}\right)+A_{ijl}(\omega,\mathbf{K})E_l$$

$$\delta\varepsilon_{ij}\left(E\right)=\begin{bmatrix}0 & AE_z & 0\\ AE_z & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Linear birefringence

2) Interface mechanism

Bloch functions for the valence band Γ_8 are

$$\left|\Gamma_8, 3/2\right\rangle = \frac{1}{\sqrt{2}}(X + iY)\uparrow$$

$$\left|\Gamma_8, 1/2\right\rangle = \frac{1}{\sqrt{6}}\left[2Z\uparrow - (X + iY)\uparrow\right]$$

$$\left|\Gamma_8, -1/2\right\rangle = \frac{1}{\sqrt{6}}\left[2Z\downarrow + (X - iY)\downarrow\right]$$

$$\left|\Gamma_8, -3/2\right\rangle = \frac{1}{\sqrt{2}}(X - iY)\downarrow$$

Boundary conditions

We can expand an arbitrary wave function in the heterostructure in the set of these functions

$$\Psi = \sum_{i=1} F_i(r) \left| \Gamma_8, i \right\rangle$$

In a heterostructure we need boundary conditions

1). Continuity of the wavefunction $\mathbf{F}_A = \mathbf{F}_B$

2). Continuity of flax $\left(\hat{v}_z \mathbf{F}\right)_A = \left(\hat{v}_z \mathbf{F}\right)_B$

$\hat{\mathbf{v}}$ - Velocity operator $\hat{v} \equiv \frac{1}{\hbar} \frac{\partial \hat{H}}{\partial K}$

Symmetry of a normal quantum well

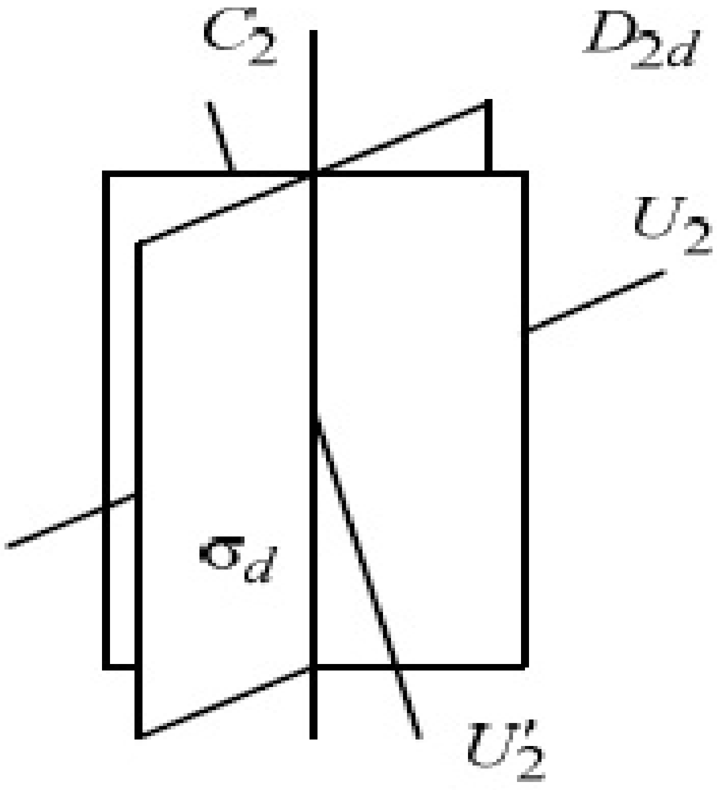
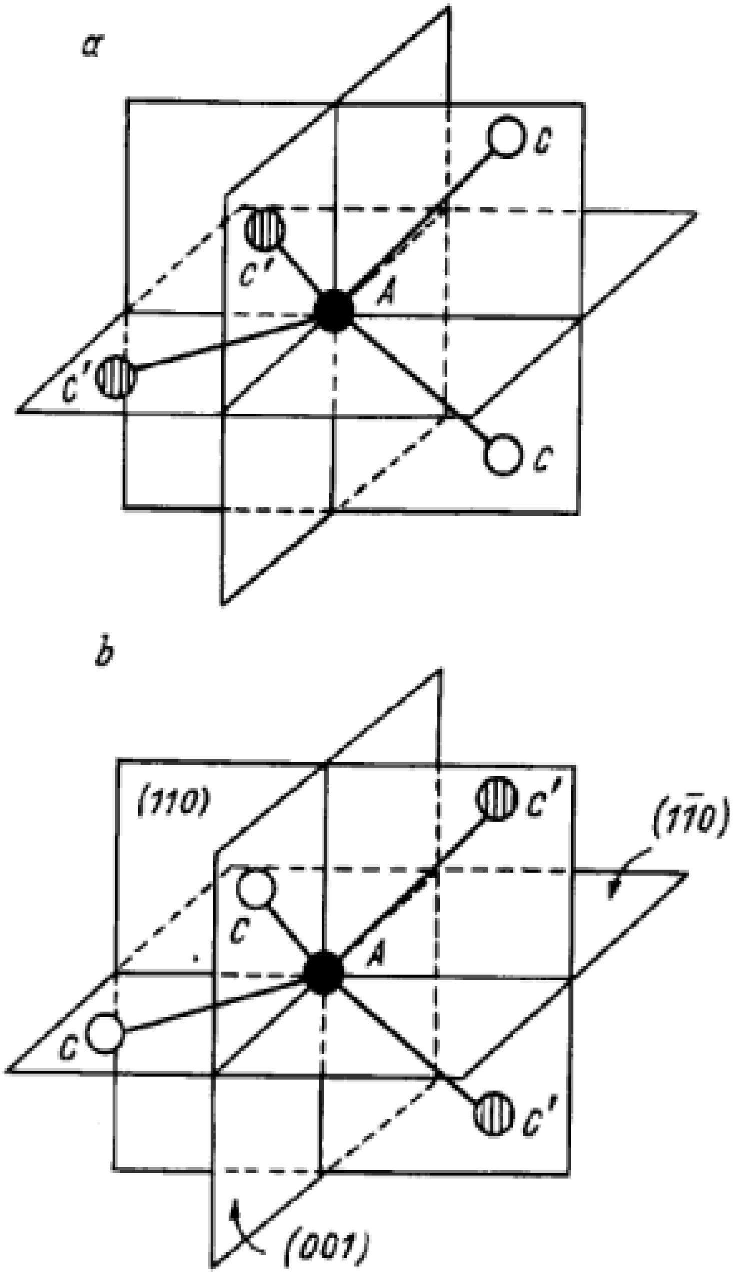


Рис. 4.5. Группа D_{2d}

$$D_{2d} : C_2, 2S_4, 2U_2, 2\sigma_d$$



Symmetry of a real interface In QWs based on zinc blend type semiconductors

$$C_{2v} : C_2, \sigma_1, \sigma_2$$

Boundary conditions taking into account low interface symmetry for valence band Γ_8

$$\left(F_j\right)_A = \left(F_j\right)_B$$

$$\left(\nabla^j F_j\right)_A = \left(\nabla^j F_j\right)_B + \frac{2}{\sqrt{3}} t_{LH} \left\{J_x J_y\right\}_{jj'} F_{j'}$$

$$\nabla^{\pm 3/2} \equiv a_0 \frac{m_0}{m_{hh}} \frac{\partial}{\partial z}, \quad \nabla^{\pm 1/2} \equiv a_0 \frac{m_0}{m_{lh}} \frac{\partial}{\partial z}$$

Wave function for electrons

$$\psi^{e1}_{\pm 1/2} = K(\mathrm{z}) \left| \Gamma_6, \pm 1/2 \right\rangle$$

Hole wavefunction from the boundary conditions

$$\psi^{hh1}_{\pm 3/2} = F(\mathrm{z}) \left| \Gamma_8, \pm 3/2 \right\rangle \pm i G(\mathrm{z}) \left| \Gamma_8, \mp 1/2 \right\rangle$$

Inside the well

$$F(z) = A\cos k_h z + B\sin k_h z$$
$$G(z) = C\cos k_l z + D\sin k_l z$$

In barriers

$$F(z) = F(\pm a/2)\exp\left[-\kappa_h(|z| - a/2)\right]$$
$$G(z) = G(\pm a/2)\exp\left[-\kappa_h(|z| - a/2)\right]$$

here

$$k_h = \left(2m_{hh}^A \mathcal{E}/\hbar^2\right)^{1/2}, k_l = \left(2m_{lh}^A \mathcal{E}/\hbar^2\right)^{1/2}$$
$$\kappa_h = \left[2m_{hh}^B(V - \mathcal{E})/\hbar^2\right]^{1/2}, \quad \kappa_l = \left[2m_{lh}^B(V - \mathcal{E})/\hbar^2\right]^{1/2}$$

Satisfying the boundary conditions we found $F(z)$ and $G(z)$

Transitions $(e1, -1/2; hh1, +3/2)$ and $(e1, +1/2; hh1, -3/2)$ are optically allowed

Matrix element of the transition in linear polarization

$$\left|M_{-1/2,3/2}(\mathbf{e})\right|^2 = \left|M_{1/2,-3/2}(\mathbf{e})\right|^2 = M_0^2\left(I_1^2 + \frac{1}{3}I_2^2 + \frac{2}{\sqrt{3}}I_1 I_2 \cos 2\phi\right)$$

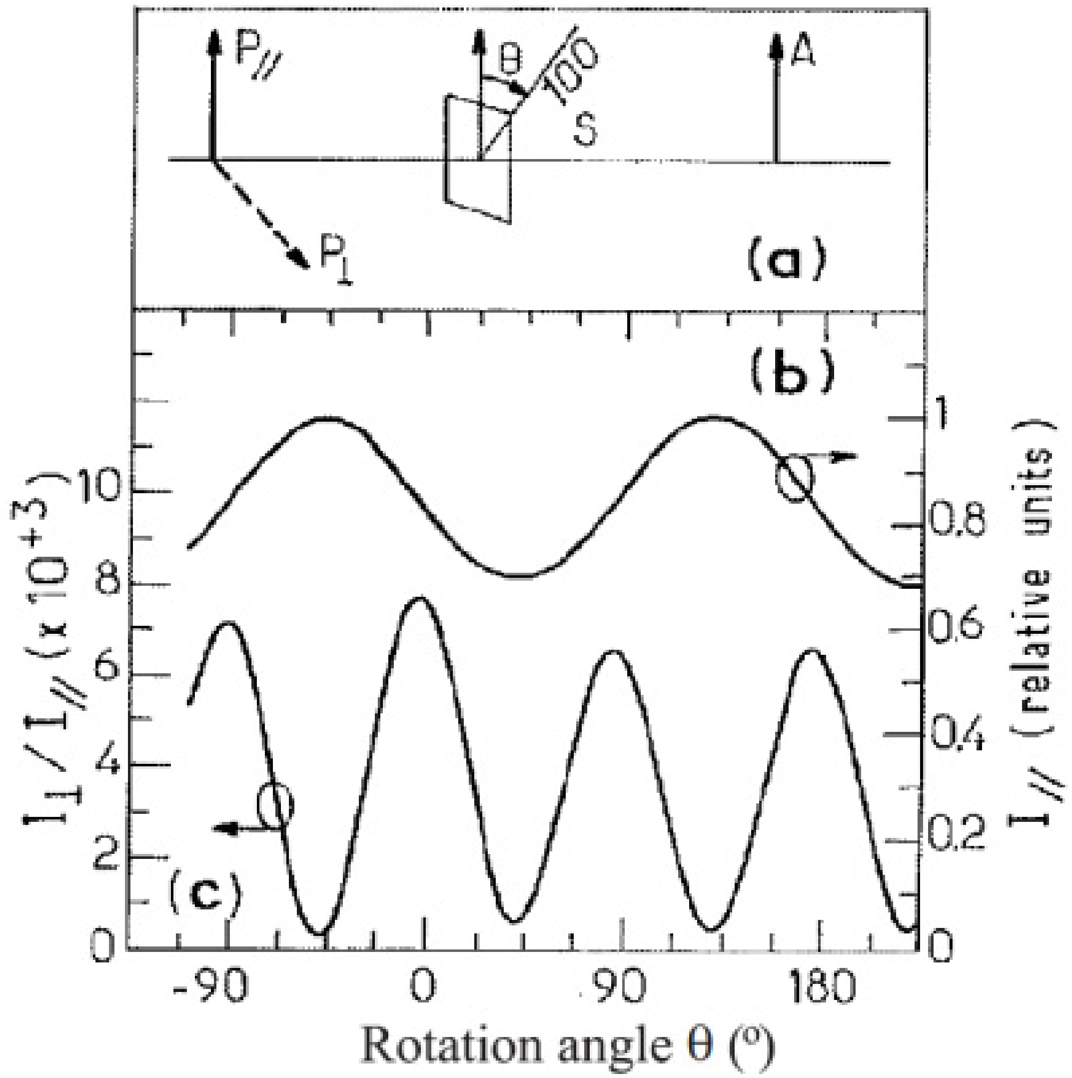
$$\phi = 0, (\mathbf{e} \parallel (110)), \quad \phi = \pi/2, (\mathbf{e} \parallel (1-10))$$

$$I_1 = \int K(z)F(z)dz \qquad I_2 = \int K(z)G(z)dz$$

Degree of linear polarization

$$\rho = \frac{\alpha_{(110)} - \alpha_{(1-10)}}{\alpha_{(110)} + \alpha_{(1-10)}} = \frac{2}{\sqrt{3}} \frac{I_1 I_2}{I_1^2 + (I_2^2/3)} \approx \frac{2}{\sqrt{3}} \frac{I_2}{I_1}$$

O. Krebs, W. Seidel, J. P. Andr´e, D. Bertho, C. Jouanin, P. Voisin:
Semicond.Sci. Technol. **12, 938 (1997).**



InGaAs(45A)/InP(68A)

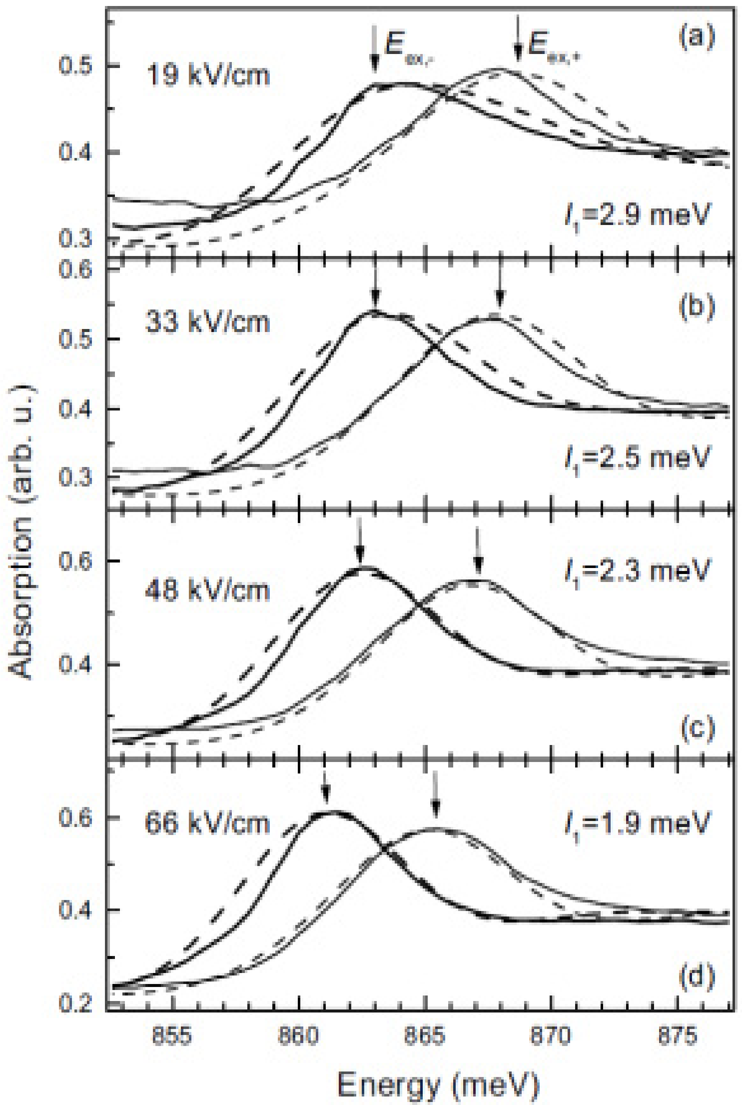
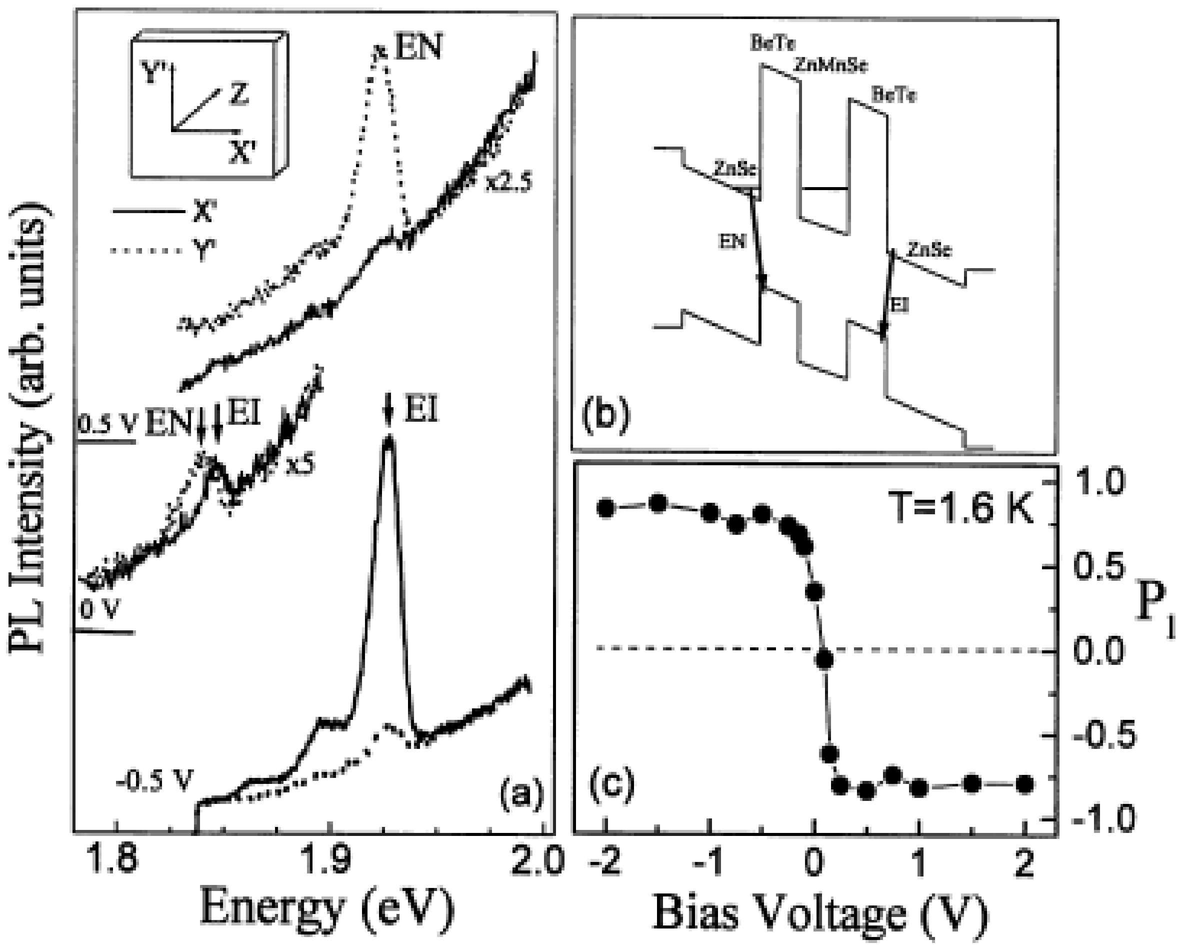


Fig. 3.13. Polarization-resolved absorption spectra of the $In_xGa_{1-x}As/InP$ sample in the region of $e1$-$hh2$ and $e1$-$lh1$ transitions. Figures (a)-(d) correspond to different fields. The incident light is polarized along the [110] (gray curves) and [$\bar{1}$10] (black curves) eigenaxes of the sample. Continuous lines display experimental data, whereas simulation results are represented by dashed lines. [3.69]

A.A. Toropov, E.L. Ivchenko, O. Krebs, S. Cortez, P. Voisin, J.L. Gentner:
Phys. Rev. **B 63, 35302 (2000). (1992).**

Giant Pockels effect



Giant quantum confined Pockels effect

Due to interface anisotropy

summary

1). Orbital magneton is 10 times larger the spin ones.
2). In Mixing of the center of mass motion and relative
motion cal strongly modify Zeeman effect and diamagnetic shift.
3). In the structures without inversion symmetry there is effect
of "parity".
4). Electric field can leads to localization.
5). Giant Pockels effect due to interfaces.

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