Lection 4

3). <u>Optics of excitons in nanostructures</u> (4h)
(exciton -photon interaction in an array of QWs; short-period SR; Bragg QW structures, diffraction from an array of QWW and QD; polaritons in microcavities)

Periodical structures with QWs

Let the period of the structure is so that the exciton wavefunctions in neighbor well are not overlap. In this case exciton contribution to the polarization is a sum over separate wells

$$P_{exc}(z) = \sum_n P_{exc}^{(n)}(z)$$

here $P_{exc}^{(n)}(z) = \frac{1}{4\pi} \int \chi_n(z, z', \omega) E(z') dz'$ Consider <u>only lowest state</u>

Electromagnetic field induce exciton polarization

$$P_{exc}^{(n)}(z) = \frac{\pi a_B^3 \varepsilon_b \omega_{LT}}{4\pi} \int \frac{\Phi_n^*(z')\Phi_n(z)}{\omega_0^n - \omega - i\Gamma} E(z') dz'$$

here $\Phi(z) = \varphi(0, z, z), \quad \Phi_n(z) = \Phi(z - nd)$

Exciton polarization in turn, induced electromagnetic field

$$E(z) = 2\pi i \frac{k_0^2}{k} \sum_n \int dz' e^{ik|z-z'|} P_{exc}^{(n)}(z')$$

Let consider average over the QW $\quad P_n = \frac{1}{a} \int dz P_{exc}^{(n)}(z)$

Substituting E into P_{exc} we obtain equation for polarizations in different wells

$$\left(\tilde{\omega}_0 - \omega - i\Gamma\right) P_n + \sum_{n'} \Lambda_{nn'} P_{n''} = 0$$

here $\quad \Lambda_{nn'} = -i\Gamma_0 e^{ikd|n-n'|}$

This is a task for eigenwaves in a chine of oscillators with resonance frequency $\tilde{\omega}_0$ damping $\Gamma + \Gamma_0$ and coupling constants $\Lambda_{nn'}$

Substitute Bloch waves $P_n = P_0 e^{iKdn}$ for infinite array of the wells and zero damping

$$\left[\omega_0 - \omega + i\Gamma\left(1 + \frac{1}{e^{i(K+k)d} - 1} + \frac{1}{e^{i(-K+k)d} - 1}\right)\right] P_0 = 0$$

Obtain dispersion equation

$$\omega_0 - \omega - \frac{\sin kd}{\cos kd - \cos Kd} \Gamma_0 = 0$$

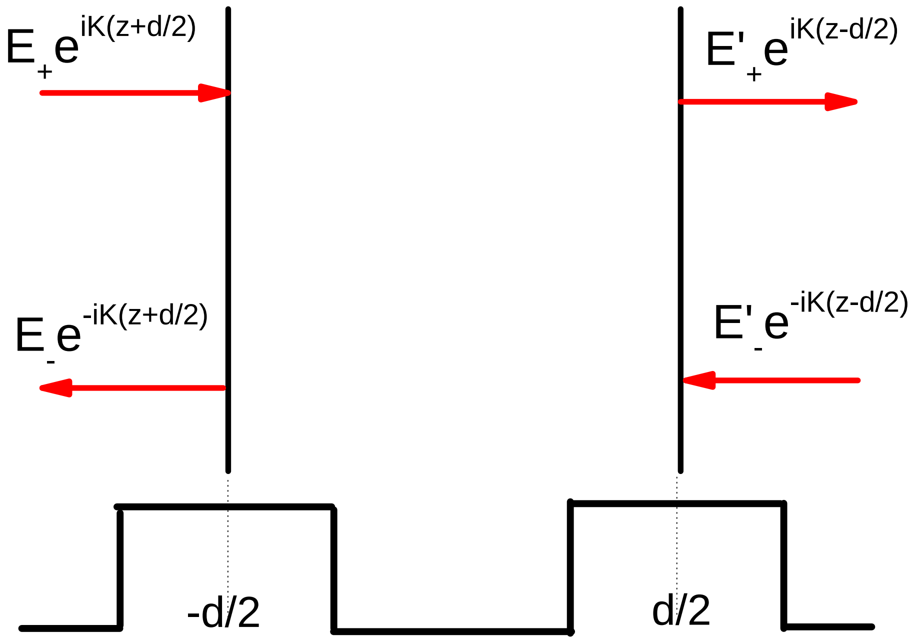
From this is clear that not all ω are available

For the periodic array of quantum wells not all waves can propagate in the structure

Method of transfer matrixes

Transfer matrix thru N layers is the product of matrixes thru each layer

$$\mathbf{T}_N = \prod_i^N \mathbf{T}_i$$



$$def: \quad \mathbf{T}\begin{bmatrix} E_+ \\ E_- \end{bmatrix} = \begin{bmatrix} E'_+ \\ E'_- \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}\begin{bmatrix} E_+ \\ E_- \end{bmatrix}$$

The amplitudes E_+, E_- and E'_+, E'_- are related to the plates $\frac{d}{2}$ and $-\frac{d}{2}$ and corresponds to $\tilde{r}, \tilde{t}$

Introduce new r, t as $\tilde{r} = re^{ikd}$ $\tilde{t} = te^{ikd}$

Let the light incident from the left $\Longrightarrow$ $E'_- = 0$

$$\begin{cases} E'_+ = T_{11}E_+ + T_{12}E_- \\ 0 = T_{21}E_+ + T_{22}E_- \end{cases}$$

$$\Downarrow$$

$$\tilde{r} = \frac{E_-}{E_+} = -\frac{T_{21}}{T_{22}}, \quad \tilde{t} = \frac{E'_+}{E_+} = \frac{T_{22}T_{11} - T_{12}T_{21}}{T_{22}}$$

Let the light incident from the right $\Longrightarrow$ $E_+ = 0$

$$\begin{cases} E'_+ = T_{12} E_- \\ E'_- = T_{22} E_- \end{cases} \quad \Rightarrow \quad \tilde{r} = \frac{E'_+}{E'_-} = \frac{T_{12}}{T_{22}}, \quad \tilde{t} = \frac{E_-}{E'_-} = \frac{1}{T_{22}}$$

Taking into account $T_{11}T_{22} - T_{12}T_{21} = 1$ consequently

$$\mathbf{T} = \frac{1}{\tilde{t}} \begin{bmatrix} \tilde{t}^2 - \tilde{r}^2 & \tilde{r} \\ -\tilde{r} & 1 \end{bmatrix}$$

We express the transfer matrix in terms of reflectivity and transmission coefficients

Infinite chain of QWs

Bloch solutions (with period d) satisfy

$$\begin{bmatrix} E'_+ \\ E'_- \end{bmatrix} = e^{iKd} \begin{bmatrix} E_+ \\ E_- \end{bmatrix}$$

From the other hand

$$\begin{bmatrix} E'_+ \\ E'_- \end{bmatrix} = \mathbf{T} \begin{bmatrix} E_+ \\ E_- \end{bmatrix} \Rightarrow$$

$$\det \begin{bmatrix} T_{11} - e^{iKd} & T_{12} \\ T_{21} & T_{22} - e^{iKd} \end{bmatrix} = 0$$

and $\Rightarrow$

$$\cos Kd = \frac{T_{11} + T_{22}}{2} = \frac{\tilde{t}^2 - \tilde{r}^2 + 1}{2\tilde{t}} = \frac{e^{ikd}\left(t^2 - r^2\right) + e^{-ikd}}{2\tilde{t}}$$

$$\cos kd + i\frac{r}{1+r}\sin kd$$

Dispersion equation for polaritons in the infinit Array of quantum wells

$$\cos Kd = \cos kd - \frac{\Gamma_0}{\omega_0 - \omega - i\Gamma}\sin kd$$

It is clear that there are forbidden and allowed bands

Optical band structure

Let $\Gamma = 0$

$$\omega = \omega_0 - \frac{\sin kd}{\cos kd - \cos Kd}\Gamma_0$$

Forbidden band between

$$\omega = \omega_0 - \Gamma_0 \, tg \frac{kd}{2}, \quad at \;\; K = \frac{\pi}{d}$$

$$\omega = \omega_0 + \Gamma_0 \, tg \frac{kd}{2}, \quad at \;\; K = 0$$

Edges of bands are at:

$$Kd = \frac{\pi}{4}, \quad \frac{\pi}{2}, \quad \frac{3}{4}\pi \qquad \omega_0 + \left(1 \pm \sqrt{2}\right)\Gamma_0, \quad \omega_0 \pm \Gamma_0, \quad \omega_0 + \left(\pm\sqrt{2} - 1\right)\Gamma_0$$

Short period superlattices $Kd << 1$

$$1-\frac{1}{2}(Kd)^2 \approx 1-\frac{1}{2}(kd)^2 - kd\,\frac{\Gamma_0}{\omega_0-\omega-i\Gamma}$$

From the other hand $\left(\frac{cK}{\omega}\right)^2 = \varepsilon_{eff}(\omega)$

$$\varepsilon_{eff}(\omega) = \varepsilon_0 + \frac{\varepsilon_0 \omega_{LT}^{MQW}}{\omega_0-\omega-i\Gamma} \quad \Rightarrow \quad \omega_{LT}^{MQW} = 2\Gamma_0 / kd$$

In this case we can use effective media approximation

Finite array of QWs

(Weak exciton photon interaction $\Gamma_0 << |\omega_0 - \omega - i\Gamma|$)

$$r_N = e^{ikd}\left(1 + e^{2ikd} + e^{4ikd} + \dots\right) r = e^{ikNd} \frac{\sin Nkd}{\sin kd} r$$

$$R_N(\omega_0) = \left(\frac{\sin Nk(\omega_0)d}{\sin k(\omega_0)d} \right)^2 r^2(\omega_0), \quad r(\omega_0) = -\frac{\Gamma_0}{(\Gamma_0 + \Gamma)}$$

Arbitrary exciton photon interaction

one can obtain

$$r_N = \frac{\tilde{r}\sin NKd}{\sin NKd - \tilde{t}\sin(N-1)Kd}$$

$$t_N = \frac{\tilde{t}\sin Kd}{\sin NKd - \tilde{t}\sin(N-1)Kd}$$

here $\tilde{r} = re^{ikd}, \quad \tilde{t} = te^{ikd}$

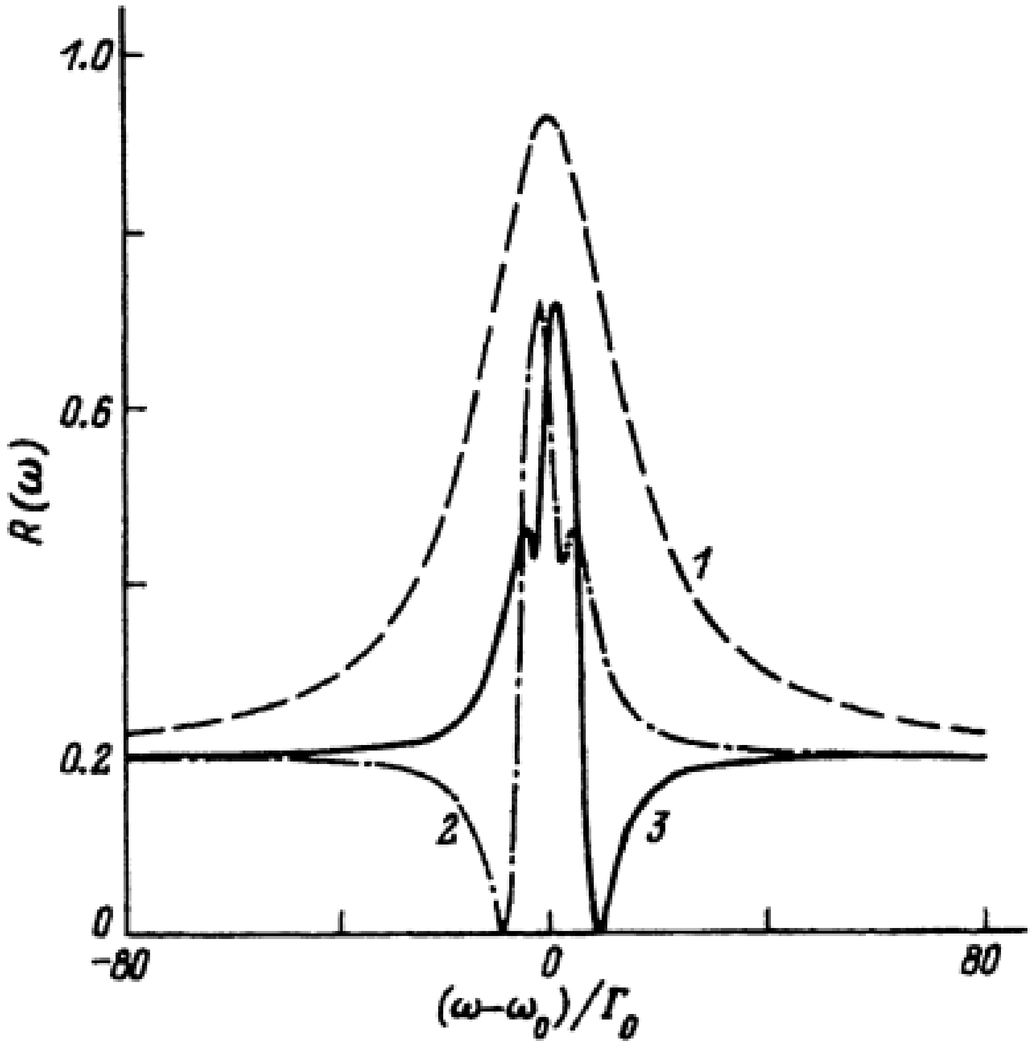
One can see that there is a special case $Kd = \pi$

Bragg quantum well structures $Kd = \pi$

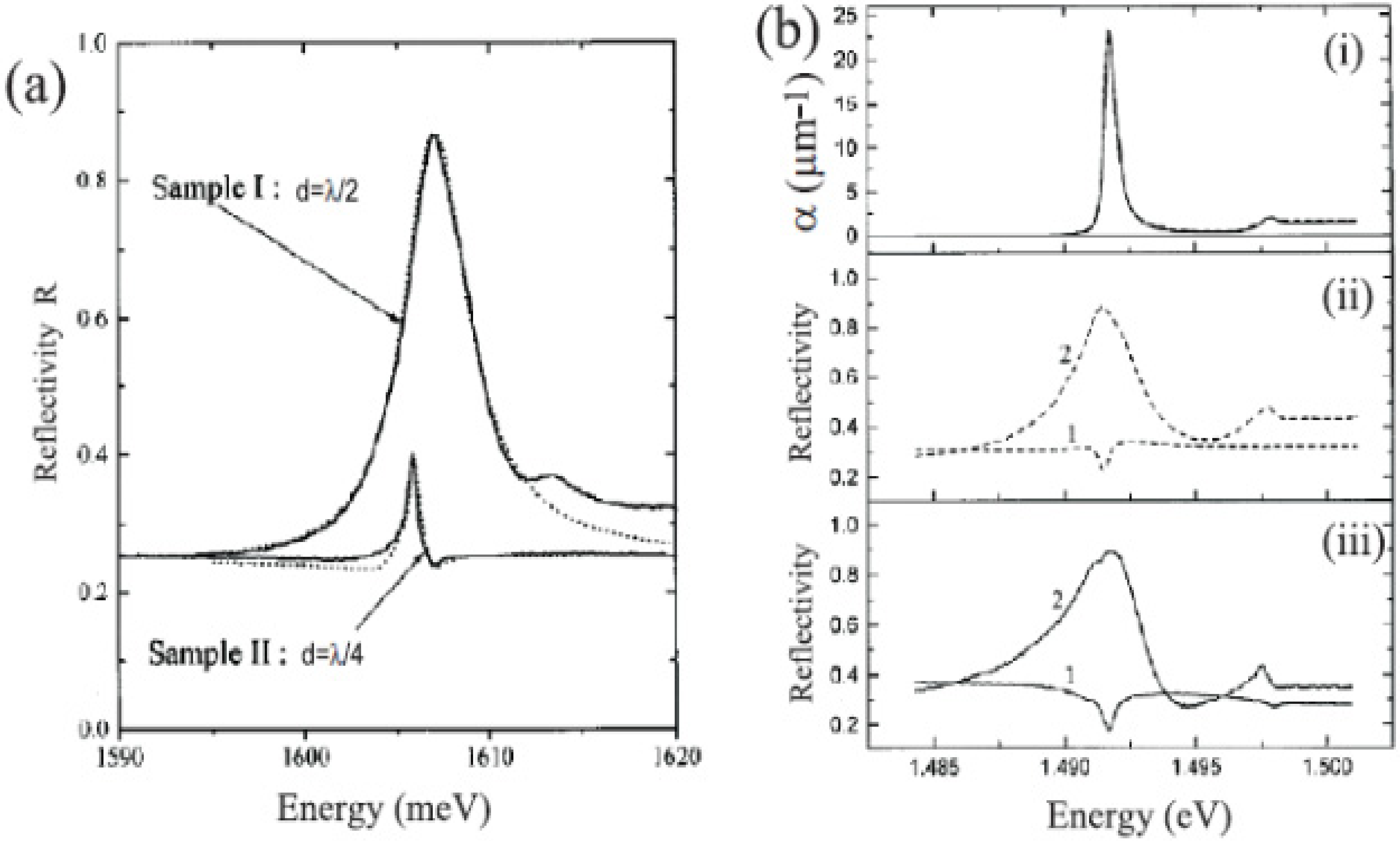
$$r_N = -\frac{iN\Gamma_0}{\omega_0 - \omega - i\left(\Gamma + N\Gamma_0\right)}$$

$$t_N = \left(-1\right)^N \frac{\omega_0 - \omega - i\Gamma}{\omega_0 - \omega - i\left(\Gamma + N\Gamma_0\right)}$$

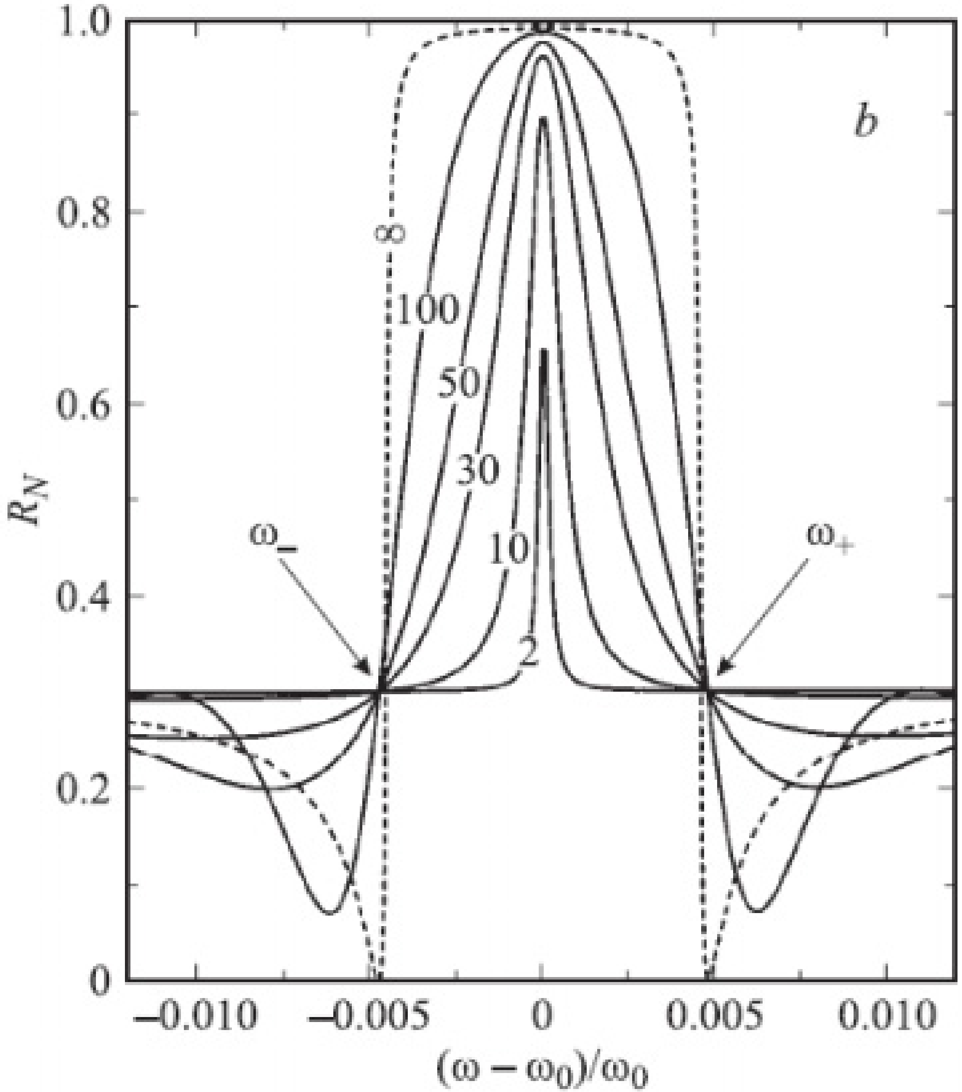
Bragg quantum well structures



Bragg quantum well structures, experiment



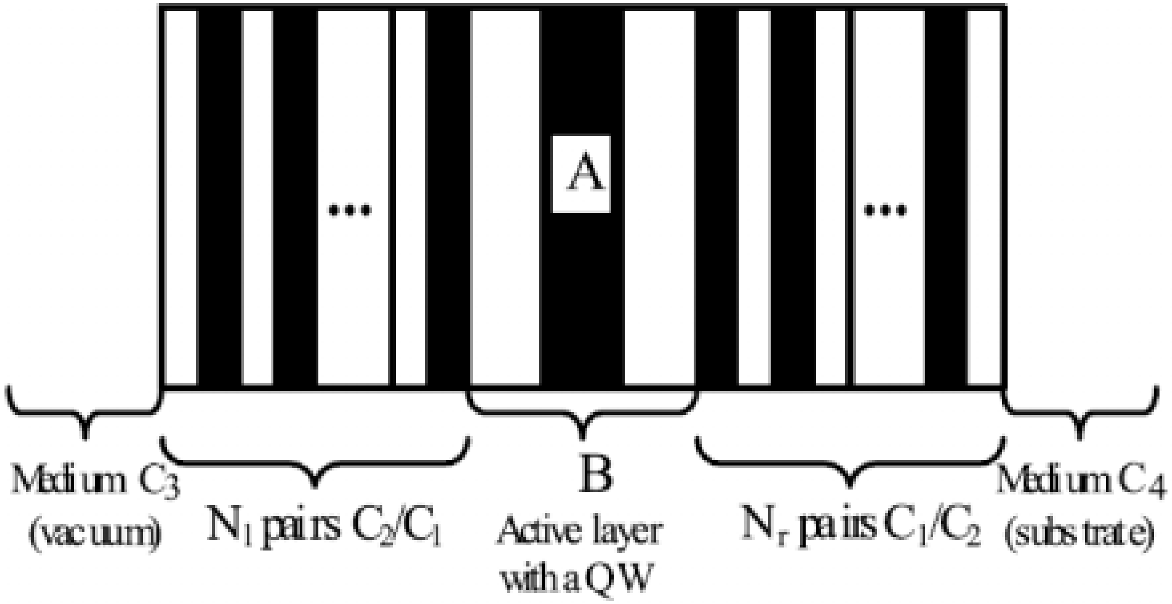
Bragg QWs.
Photonic band-gap versus super radiance



Microcavity

Layers c_1 / c_2 refractive indexes n_1 / n_2 thickness a_1 / a_2

$n_2 > n_1$



$\tilde{\lambda}/2$ cavity $\qquad \tilde{\lambda} = 2\pi\left(\dfrac{c}{\tilde{\omega} n_b}\right)$

layers $\qquad k_1 a_1 = k_2 a_2 = n_1 \dfrac{\tilde{\omega}}{c} a_1 = n_2 \dfrac{\tilde{\omega}}{c} a_2 = \dfrac{\pi}{2}$

Microcavity

Distribution of the electromagnetic field in a microcavity

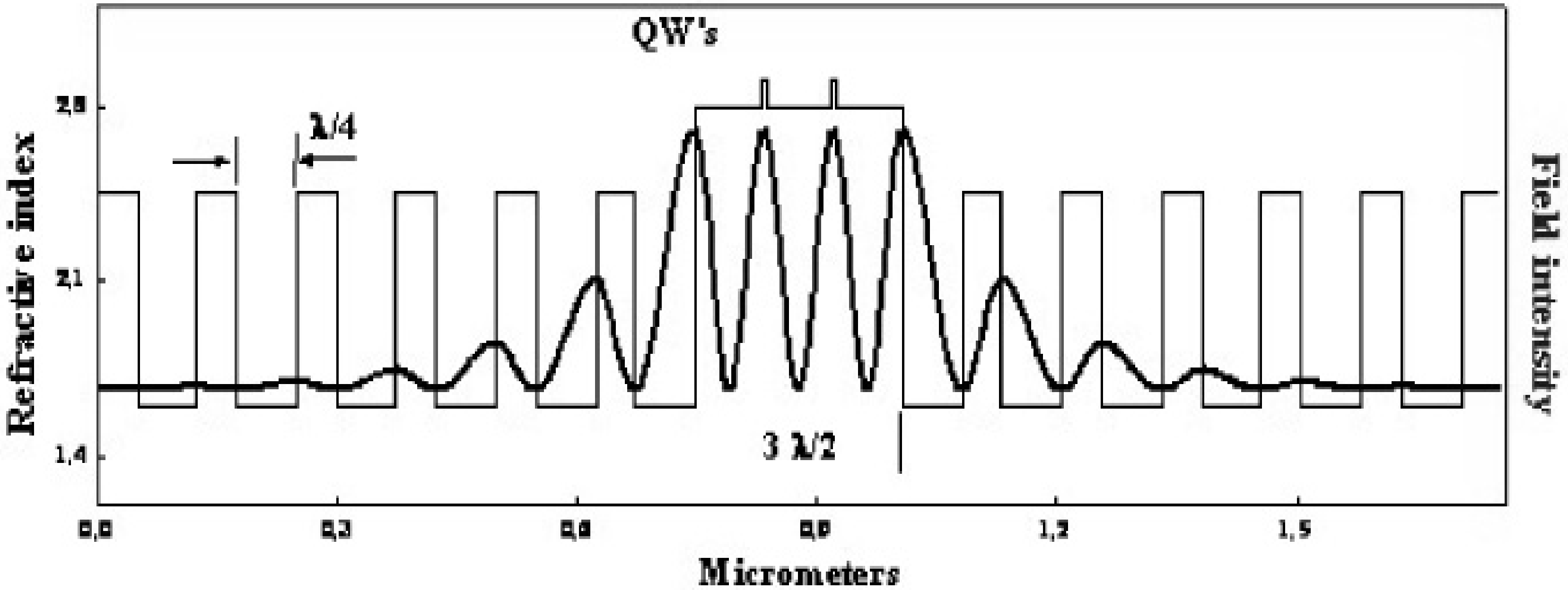
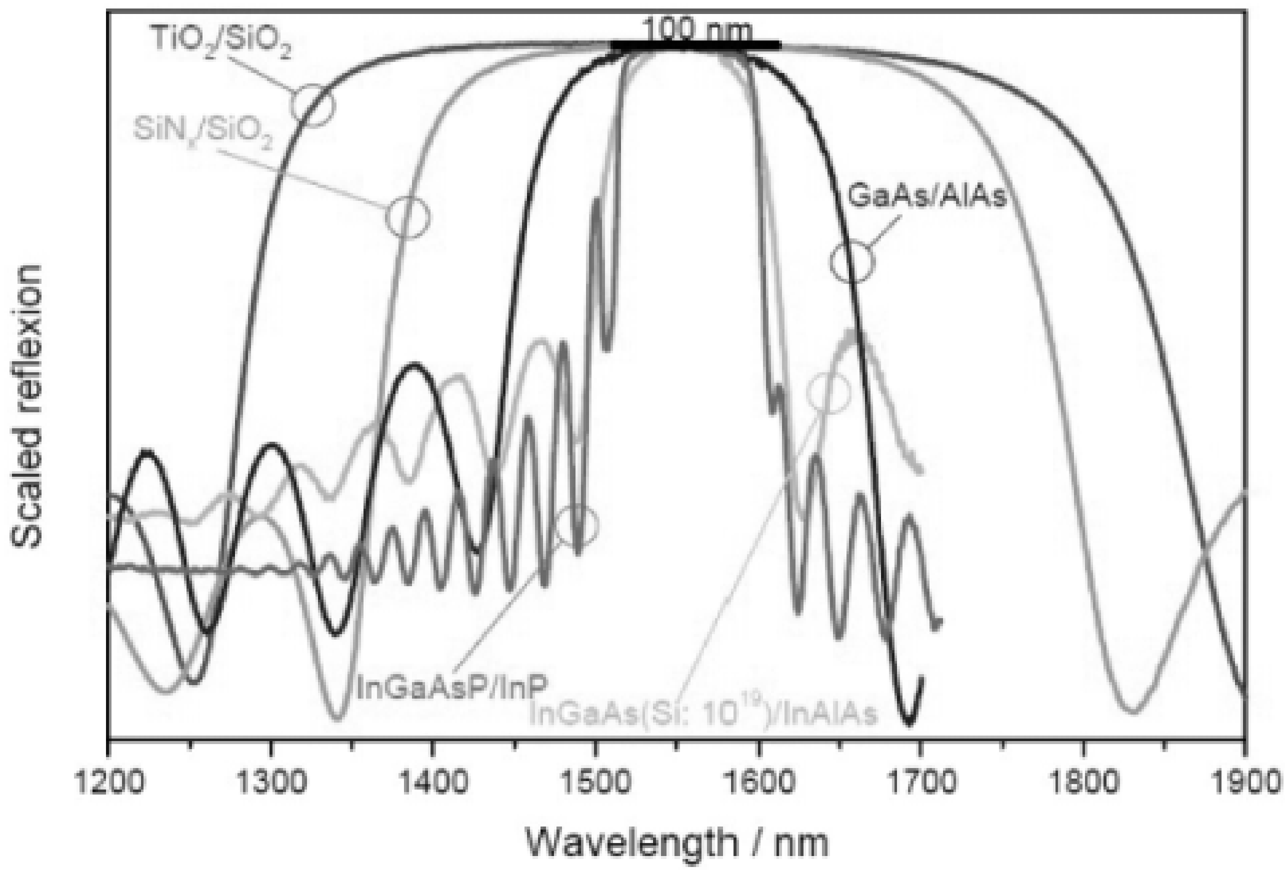


Fig. 2.11: Refractive-index profile and intensity of electric field of the eigenmode of a typical planar micro-cavity.

Reflection from the Bragg mirror

Stop-band



Two oscillator model

Electromagnetic field $E(t)$ polarization $P(t)$

Average polarization $P(t) \equiv \frac{1}{a}\int dz P(z,t)$

$$\begin{cases} \frac{d^2}{dt^2}P(t) + \omega_0^2 P(t) + 2\Gamma \frac{d}{dt}P(t) = q_1 E(t) \\ \frac{d^2}{dt^2}E(t) + \tilde{\omega}^2 E(t) + 2\gamma \frac{d}{dt}E(t) = q_2 P(t) \end{cases}$$

Looking for the solution in harmonic form

$$P(t) = Pe^{-i\omega t}, \quad E(t) = Ee^{-i\omega t}$$

Close to the exciton and cavity resonances $(\Gamma, \gamma, \omega_0 - \omega) << \omega_0, \tilde{\omega}$

solution

$$(\omega_0 - \omega - i\Gamma)P = \gamma_1 E$$
$$(\tilde{\omega} - \omega - i\gamma)E = \gamma_2 P$$

$$\gamma_i = q_i / 2\tilde{\omega}$$

here $\gamma_1 \equiv \dfrac{\varepsilon_b}{2\pi ka}\Gamma_0$

Weak exciton photon coupling

$$(\Gamma - \gamma)^2 > 4\gamma_1\gamma_2$$

$$\omega_\pm = \tilde{\omega} - i\frac{\Gamma + \gamma}{2} \pm i\tilde{\tilde{\gamma}} \qquad \tilde{\tilde{\gamma}} = \sqrt{\frac{(\Gamma - \gamma)^2}{2} - \gamma_1\gamma_2}$$

the exciton photon modes – the same resonant frequency but different damping

Strong exciton photon coupling

$$(\Gamma - \gamma)^2 < 4\gamma_1\gamma_2$$

$$\omega_{\pm} = \tilde{\omega} \pm \Omega - i\frac{\Gamma + \gamma}{2} \qquad \Omega = \sqrt{\gamma_1\gamma_2 - \frac{(\Gamma - \gamma)^2}{2}}$$

Raby splitting $\omega_+ - \omega_- = 2\Omega$

the exciton photon modes – the same damping but different resonant frequencies

Polariton effective mass in microcavity

$$E_m(\mathrm{k}) = \hbar\,\mathrm{c}|k| = \hbar c\sqrt{k_z^2 + |k_{||}|^2} \approx \hbar c\left(|k_z| + \frac{|k_{||}|^2}{2|k_z|}\right)$$

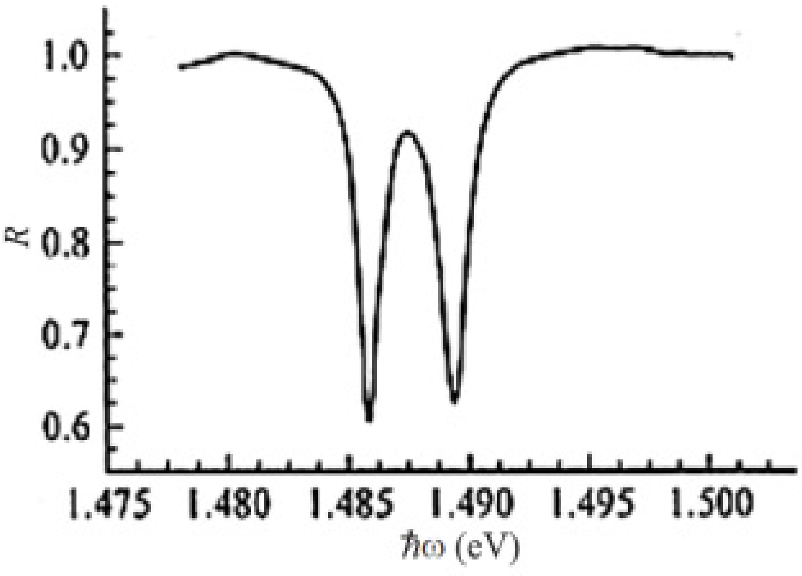
$$|k_z| = \frac{\pi m}{L}, \qquad m = 1, 2, 3, \ldots$$

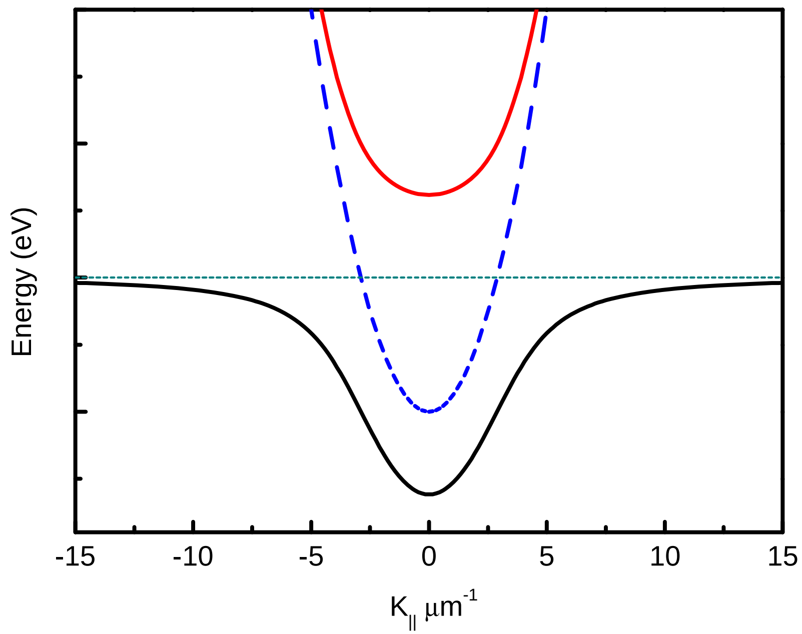
$$E(\mathrm{k}) = E_0 + \frac{\hbar^2 |k_{||}|^2}{2M_{eff}}$$

$$M_{eff} = \frac{\pi}{\hbar c L} \qquad\qquad E_0 = \hbar c \frac{\pi}{L}$$

The photon mass is of the order of $10^{-4} m_0$

Exciton and photon modes in microcavity





Parametric polariton scattering

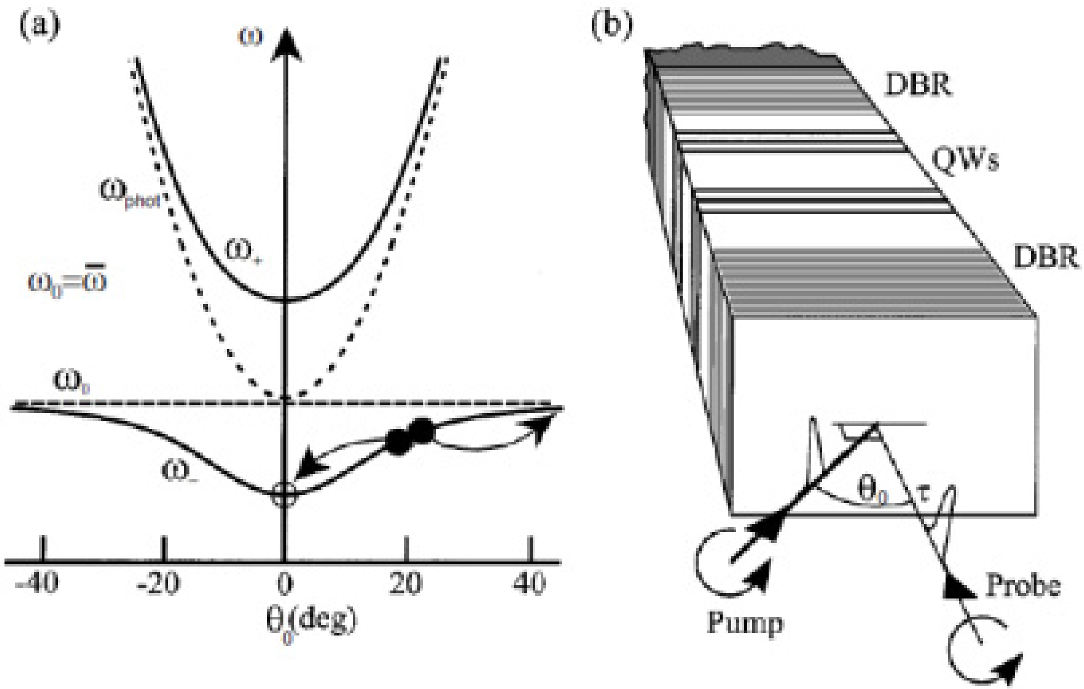
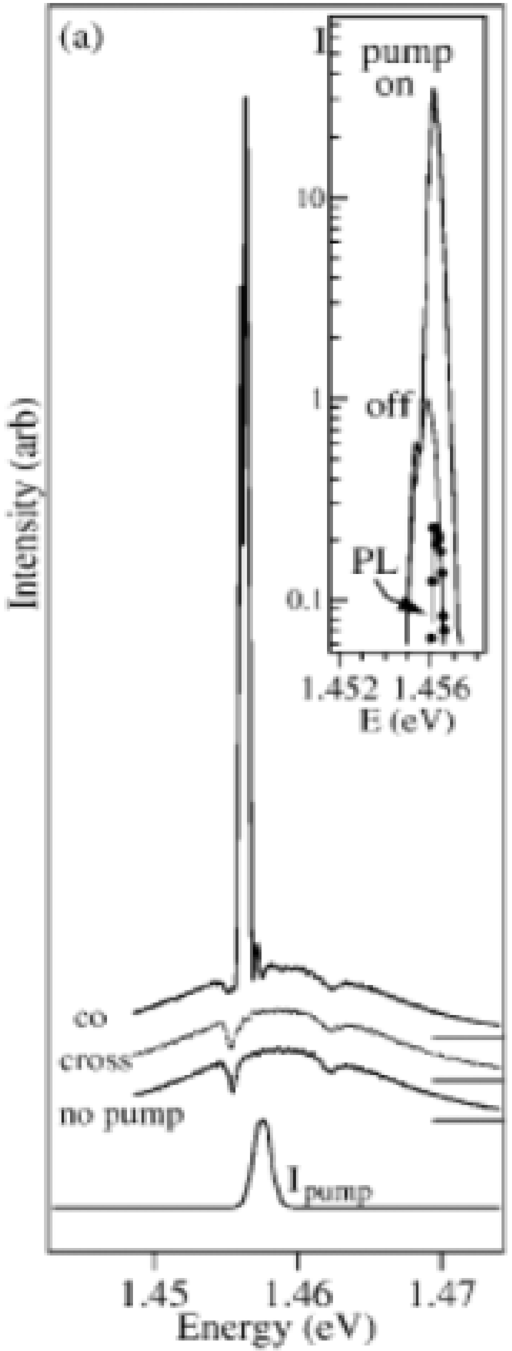


Fig. 7.6. (a) Polariton dispersion relation vs. incident angle θ_0 at zero detuning. Microcavity (exciton) frequencies ω_{phot} (ω_0) shown dashed. Probe polariton (open circle) stimulates the scattering of pump polaritons (filled circles). (b) Sample structure and experimental geometry probed at normal incidence and time delay τ, while changing the pump angle θ_0. From [7.78].



Parametric polariton amplification

Superaniance in Bragg structures